Optimized *model-based* undersampling and reconstruction for dynamic MRI based on *support splitting*.

Application to phase contrast MRI carotid blood flow imaging.

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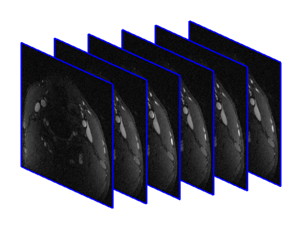
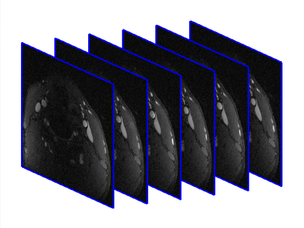
Phase contrast carotid blood velocity imaging

Two sets of images:

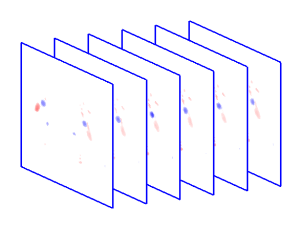


Reference frames

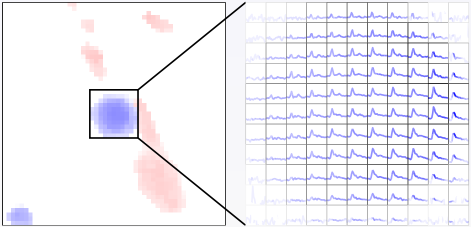


Velocity encoded frames

Velocity information: phase difference



Phase difference frames



Right carotid artery blood velocity

Phase contrast carotid blood velocity imaging

Signal acquisition: lines in the Fourier domain (k-space)

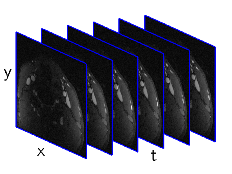
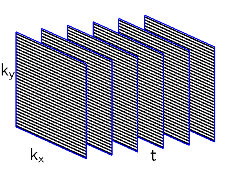
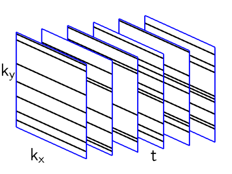


Image frames

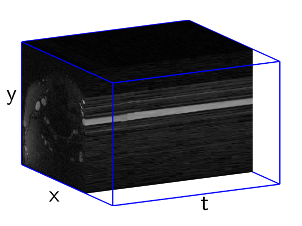


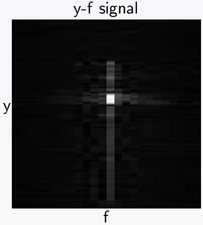
fully sampled

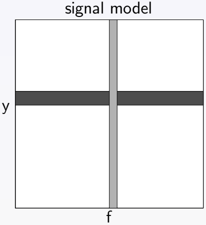


accelerated

Signal model for one line through the carotid

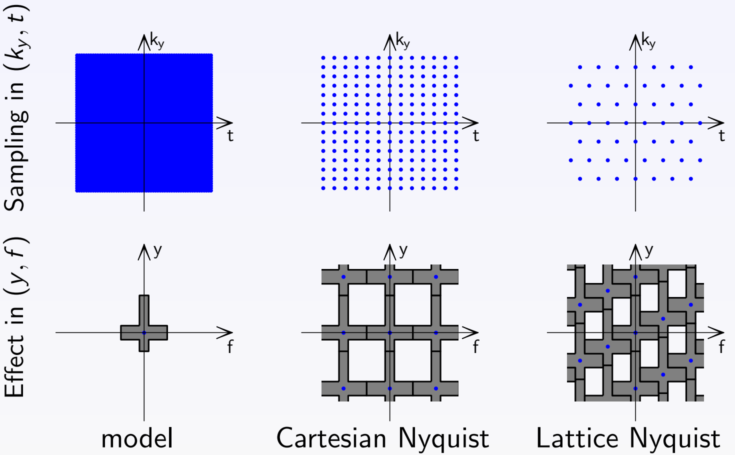






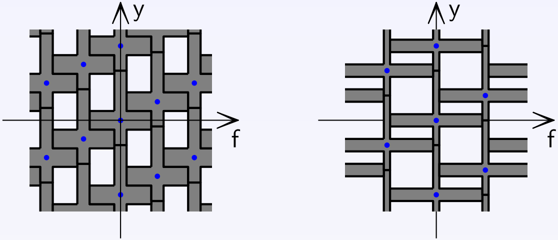
Traditional accelerated dyn-MRI (UNFOLD, PARADIGM)

Lattice sampling 2D version of classical sampling theory



Traditional accelerated dyn-MRI (UNFOLD, PARADIGM)

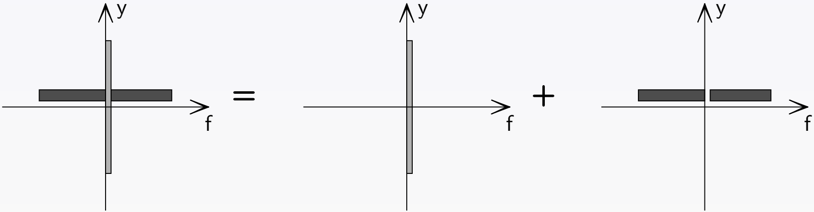
The performance of lattice sampling depends on packability



decent acceleration

poorer acceleration

Possible improvement: split the support for improved packability



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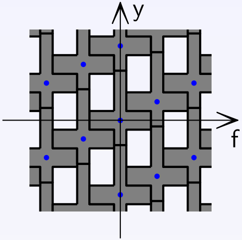
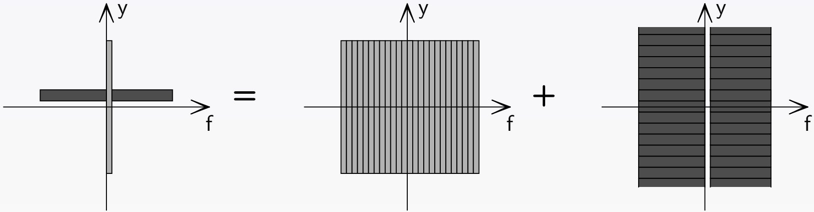


decent acceleration poorer acceleration

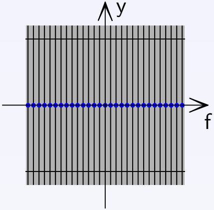
Possible improvement: split the support for improved packability

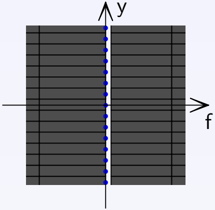


Accelerated acquisition for cross-shaped support

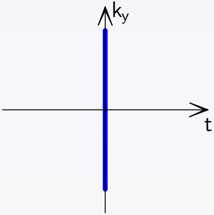
static support | **dynamic support**

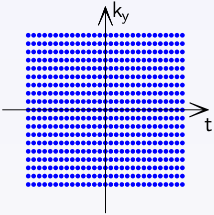
Packing





Sampling





Properties

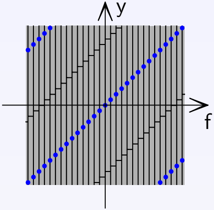
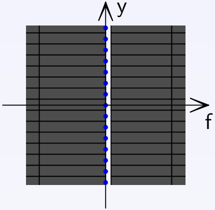
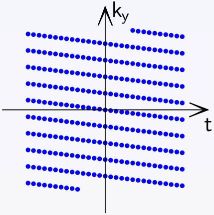
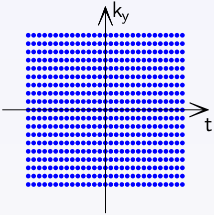
Static support:

- only DC $\rightarrow$ only one sample for each k_y
- some sampling patterns are more spread in time

Dynamic support:

- for each t the number of k_y samples must be greater than the size of the dynamic ROI

Accelerated acquisition for cross-shaped support

	static support	dynamic support
Packing		
Sampling		
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Accelerated acquisition for cross-shaped support

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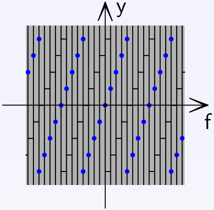
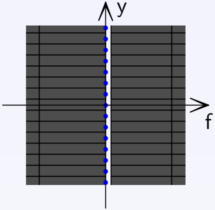
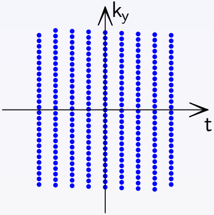
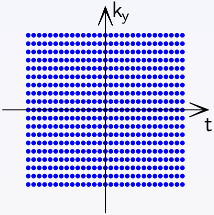
Packing		
Sampling		
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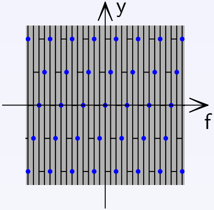
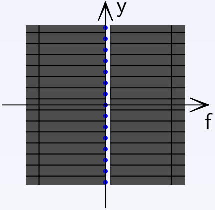
Accelerated acquisition for cross-shaped support

static support | dynamic support

Packing





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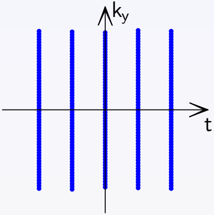

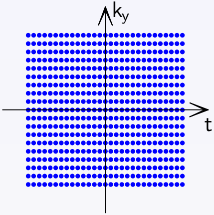

Properties

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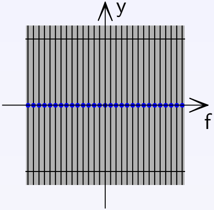
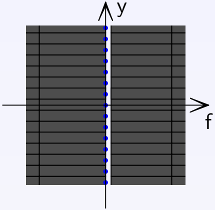
Accelerated acquisition for cross-shaped support

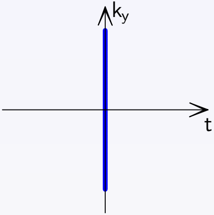
static support | **dynamic support**

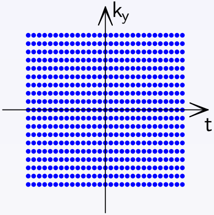
Packing





Sampling





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Accelerated acquisition for cross-shaped support

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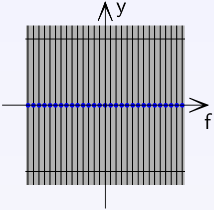
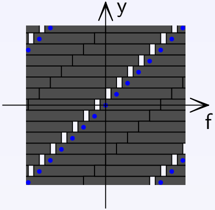
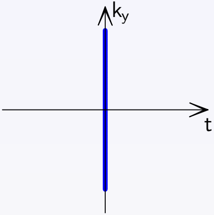
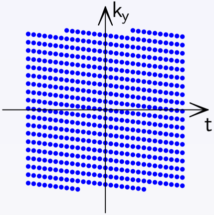
Packing		
Sampling		

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Accelerated acquisition for cross-shaped support

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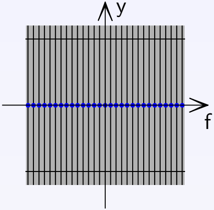
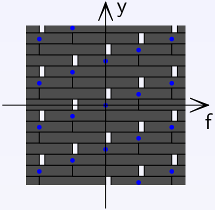
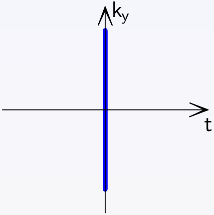
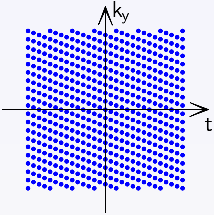
Packing		
Sampling		
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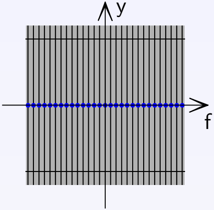
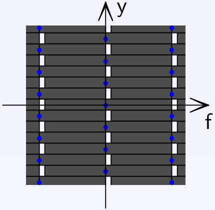
Accelerated acquisition for cross-shaped support

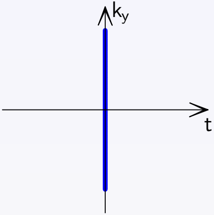
static support | dynamic support

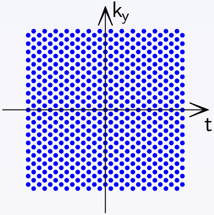
Packing





Sampling





Properties

static support

- only DC $\rightarrow$ only one sample for each k_y
- some sampling patterns are more spread in time

dynamic support

- for each t the number of k_y samples must be greater than the size of the dynamic ROI

Accelerated acquisition for cross-shaped support

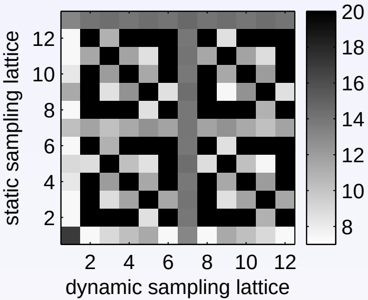
Properties of the support splitting approach

- Allows much higher acceleration factors
  $\Longrightarrow$ up to minimum sampling rate for a cross-shaped support
- Penalty in terms of conditioning for the reconstruction
  - single lattice sampling: condition number $= 1$
  - two lattices sampling: condition number > 1
- Different combinations of two lattices $\rightarrow$ different conditioning

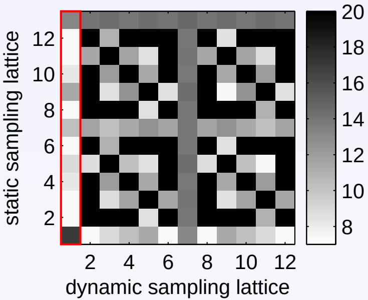
Optimization of the choice of two lattices

- *Finite number of lattices* given a support shape and image/time resolution
- conditioning can be optimized via *exhaustive search*

Optimization of the choice of two lattices



Optimization of the choice of two lattices



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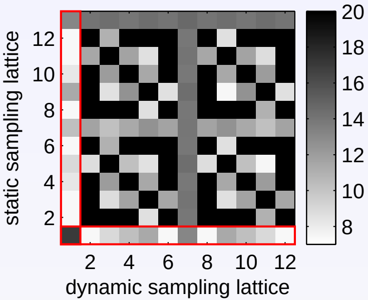

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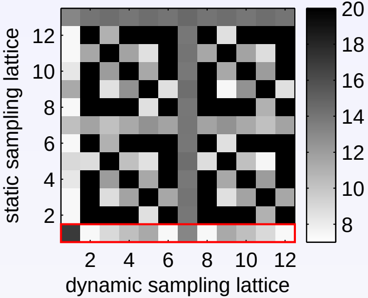
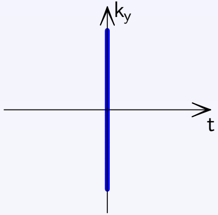


lattice 1 (static support)

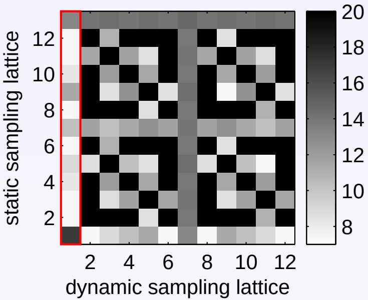


Properties

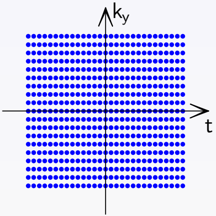
- All samples at the same time location in the cycle

$\Longrightarrow$ Not applicable for acceleration!

Optimization of the choice of two lattices



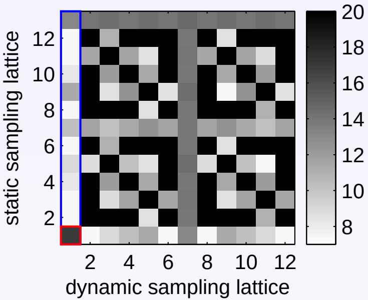
lattice 2 (dynamic support)



Properties

- Samples aligned along lines of constant k_y locations
- Can be achieved with retrospective gating

Optimization of the choice of two lattices



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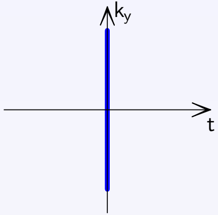
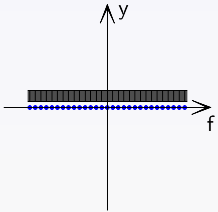


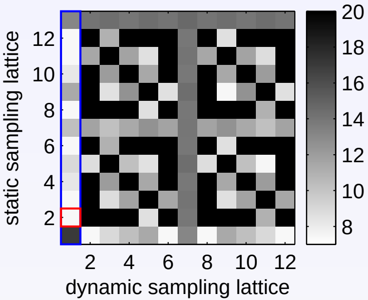
Effect on the dynamic support



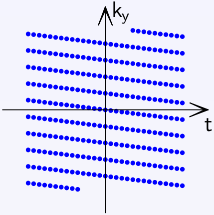
Properties

- Condition number: ≈ 17
- No spread over time
- No spread of the dyn. support

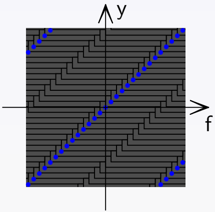
Optimization of the choice of two lattices



lattice 1 (static support)



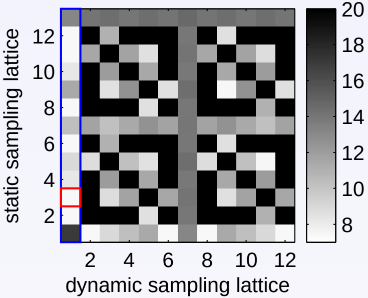
Effect on the dynamic support



Properties

- Condition number: ≈ 7.4
- Maximum spread over time
- Maximum spread of the dyn. support

Optimization of the choice of two lattices



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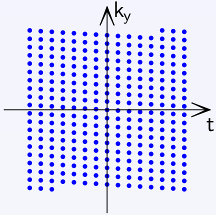
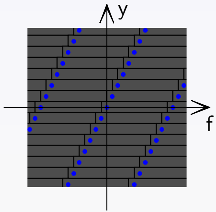


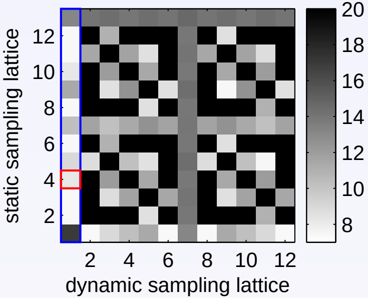
Effect on the dynamic support



Properties

- Condition number: ≈ 7.4
- Good spread over time
- Maximum spread of the dyn. support

Optimization of the choice of two lattices



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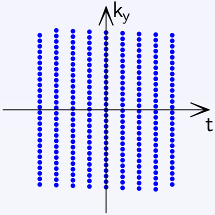
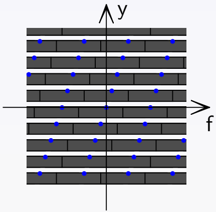


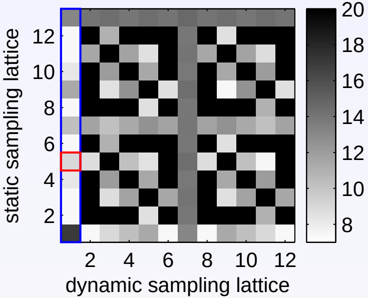
Effect on the dynamic support



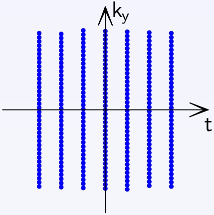
Properties

- Condition number: ≈ 8.2
- Poor spread over time
- Poor spread of the dyn. support

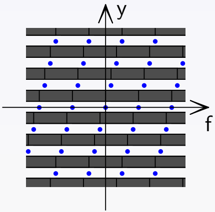
Optimization of the choice of two lattices



lattice 1 (static support)



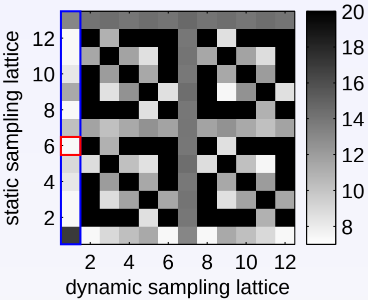
Effect on the dynamic support



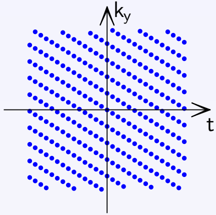
Properties

- Condition number: ≈ 9
- Poor spread over time
- Poor spread of the dyn. support

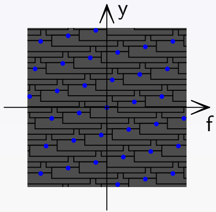
Optimization of the choice of two lattices



lattice 1 (static support)



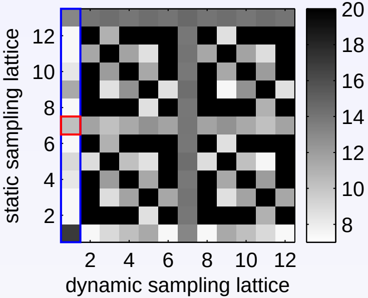
Effect on the dynamic support



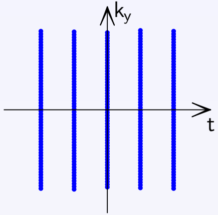
Properties

- Condition number: ≈ 7.4
- Maximum spread over time
- Maximum spread of the dyn. support

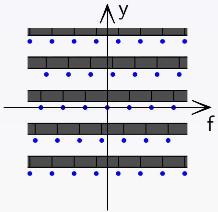
Optimization of the choice of two lattices



lattice 1 (static support)



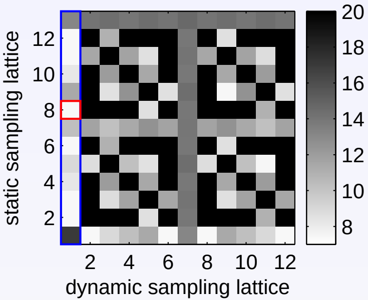
Effect on the dynamic support



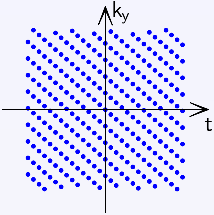
Properties

- Condition number: ≈ 10.3
- Poorer spread over time
- Poorer spread of the dyn. support

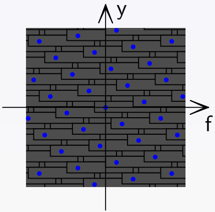
Optimization of the choice of two lattices



lattice 1 (static support)



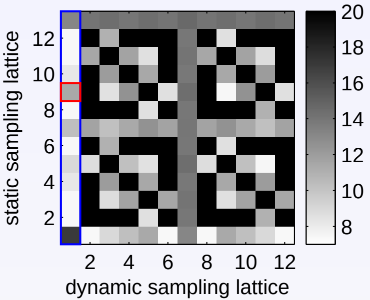
Effect on the dynamic support



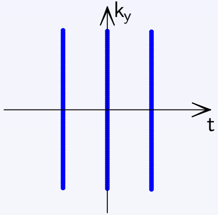
Properties

- Condition number: ≈ 7.4
- Maximum spread over time
- Maximum spread of the dyn. support

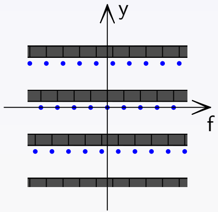
Optimization of the choice of two lattices



lattice 1 (static support)



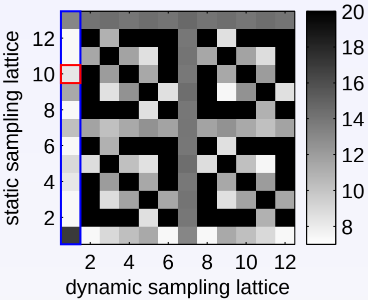
Effect on the dynamic support



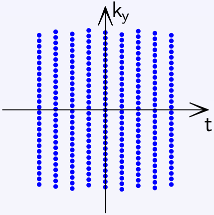
Properties

- Condition number: ≈ 11.4
- Poorer spread over time
- Poorer spread of the dyn. support

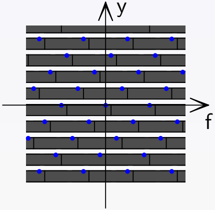
Optimization of the choice of two lattices



lattice 1 (static support)



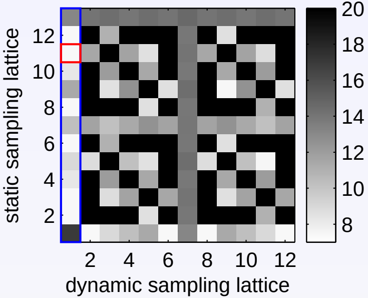
Effect on the dynamic support



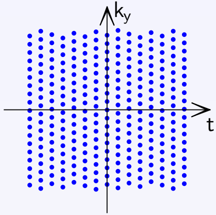
Properties

- Condition number: ≈ 8.2
- Poor spread over time
- Poor spread of the dyn. support

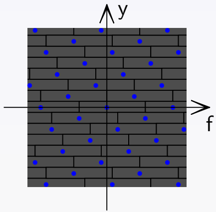
Optimization of the choice of two lattices



lattice 1 (static support)



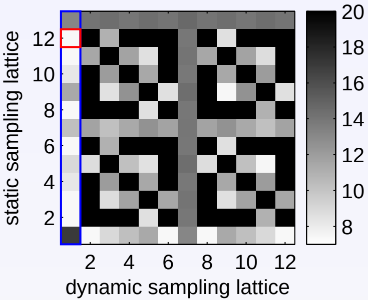
Effect on the dynamic support



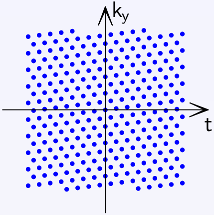
Properties

- Condition number: ≈ 7.4
- Good spread over time
- Maximum spread of the dyn. support

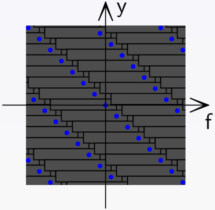
Optimization of the choice of two lattices



lattice 1 (static support)



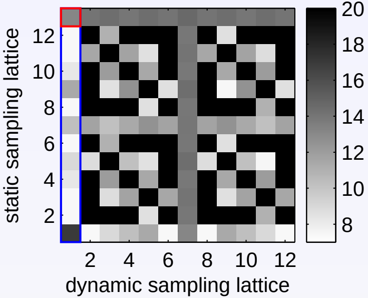
Effect on the dynamic support



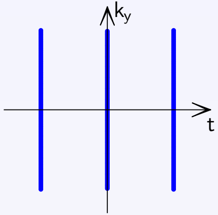
Properties

- Condition number: ≈ 7.4
- Maximum spread over time
- Maximum spread of the dyn. support

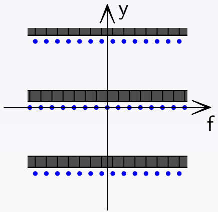
Optimization of the choice of two lattices



lattice 1 (static support)



Effect on the dynamic support



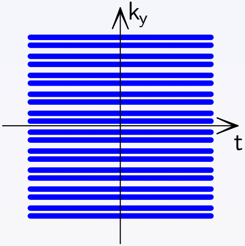
Properties

- Condition number: ≈ 13.2
- Very poor spread over time
- Very poor spread of the dyn. support

Application to carotid blood flow imaging

Detection of the dynamic support

- *Block support*: typically one block in y (or a very small number)
- The correct support should provide the best fit to the measurements in the least-squares sense
- The previous sampling pattern alone is insufficient because the number of measurements is equal to the number of coefficients to estimate
- Multi-coset sampling may be used to detect one (or several) blocks



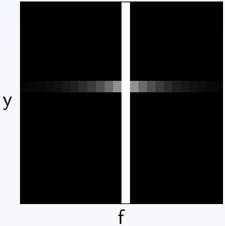
Reconstruction algorithm

1. Exhaustive search of the block-sparse dynamic support minimising the residual ($O(N^{N_b})$ possibilities for N_b blocks)
2. Global reconstruction using pseudoinverse (actually obtained in the previous step)

Application to carotid blood flow imaging — 2

Possible improvements

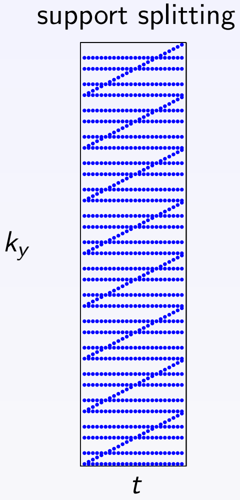
- 2D search for the dynamic support assuming a circular shape in the image
- use Wiener estimation: minimize $\mathbb{E}\,\|\hat{x} - x\|_2$ assuming
  - observation model $y = \mathbf{\Phi} x + n$ with $n \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$
  - signal model: $x \sim \mathcal{N}(0, \mathbf{R})$ with diagonal $\mathbf{R}$:

diag($\mathbf{R}$)= 

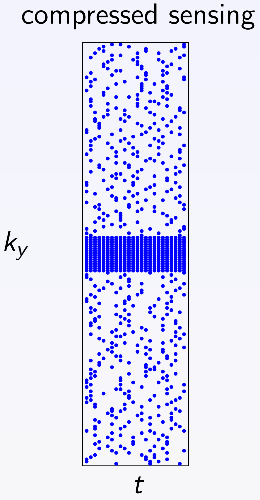
- constant static support
- exponential decay in frequency
- can be calibrated using standard pseudoinverse reconstruction

  - reconstruction: $\hat{x} = \mathbf{R}\mathbf{\Phi}^H(\mathbf{\Phi}\mathbf{R}\mathbf{\Phi}^H + \sigma^2 \mathbf{I})^{-1} y$
- use information from multiple coils (receivers)

Examples of sampling patterns (5X acceleration)



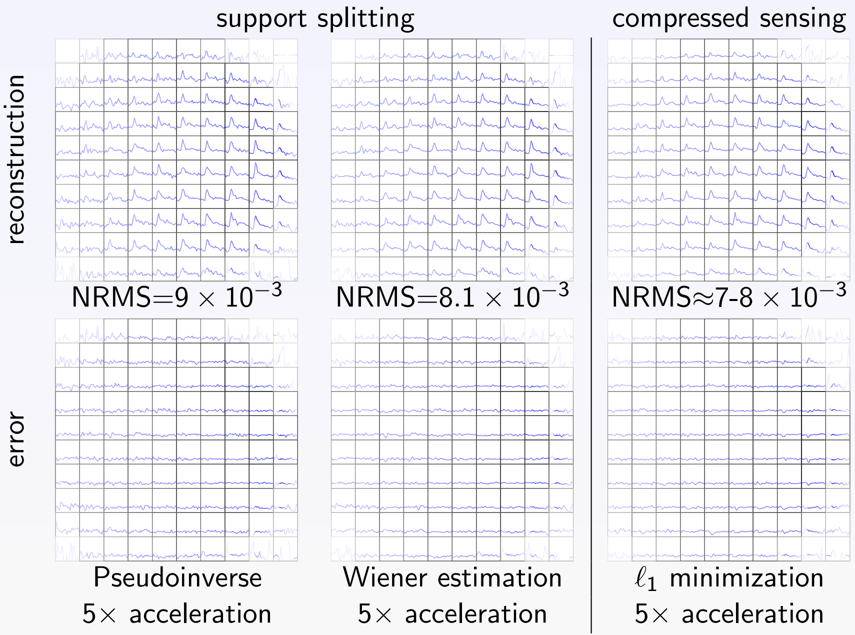
multi-coset dynamic
$+$ well-spread static
cond. ≈ 6.2



variable density random

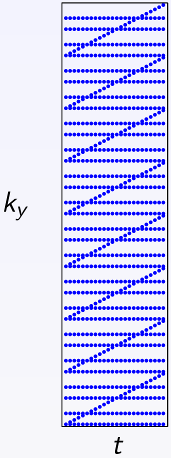
cond. $\gtrsim 200$

Examples of reconstructions (simulations)

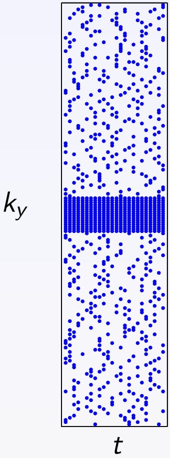


Improving the sampling pattern

support splitting

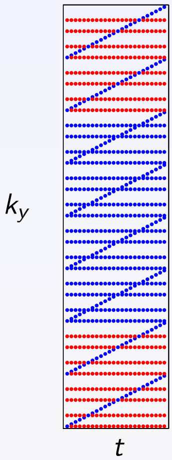


compressed sensing



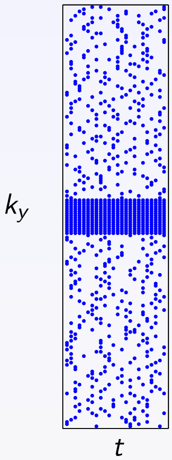
Improving the sampling pattern

support splitting

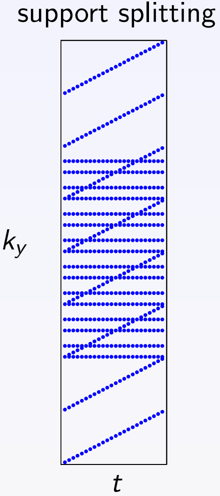


High spatial frequencies
can be assumed constant

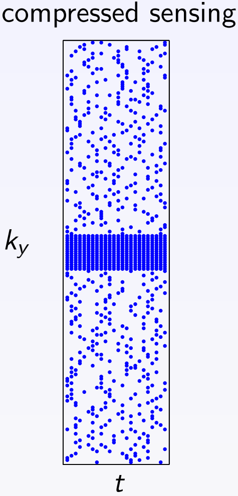
compressed sensing



Improving the sampling pattern



Reduced sampling pattern

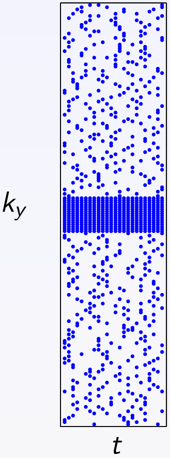


Improving the sampling pattern

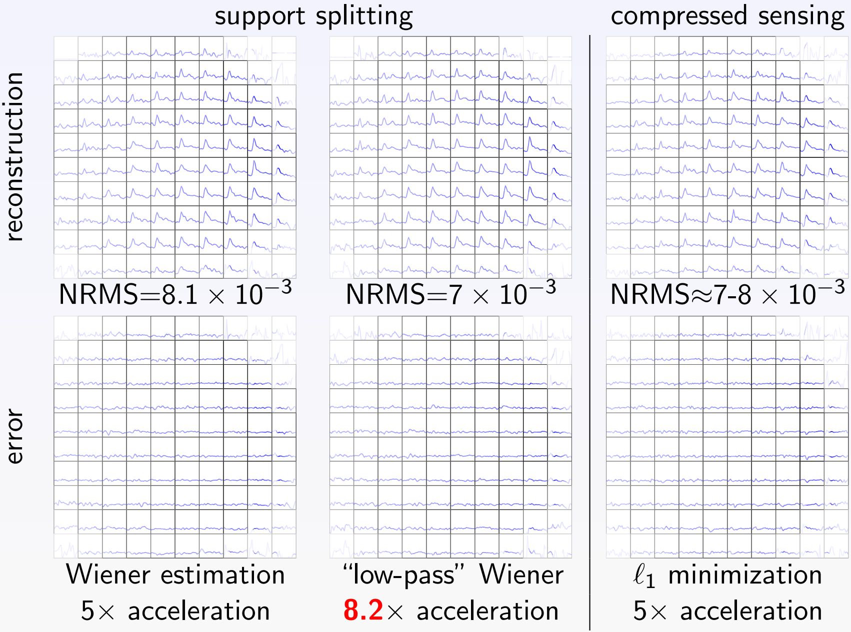
support splitting

Reduced sampling pattern with simulated high spatial frequencies

compressed sensing



Examples of reconstructions (simulations)



Conclusions

Deterministic approach based on

- support splitting
- multi lattice sampling

Properties

- Allows high acceleration factors
- Competitive with state of the art compressed sensing methods (k-t SPARSE, k-t FOCUSS,...)
- Controlled condition number
  $\Longrightarrow$ guarantees on the reconstruction quality

Applications

- Phase contrast carotid blood flow imaging
- functional MRI
- ...

Thank you for your attention