Quantitative PhotoAcoustic Tomography using Diffusion and Transport Model

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Outline

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1. Introduction
2. PhotoAcoustics
3. Quantitative PhotoAcoustic Tomography
4. Summary

Photoacoustic Imaging

- outline of photoacoustic imaging
- Photoacoustic image reconstruction
- Spectroscopic photoacoustic imaging
- Artefacts in photoacoustic imaging

Quantitative Photoacoustic Imaging

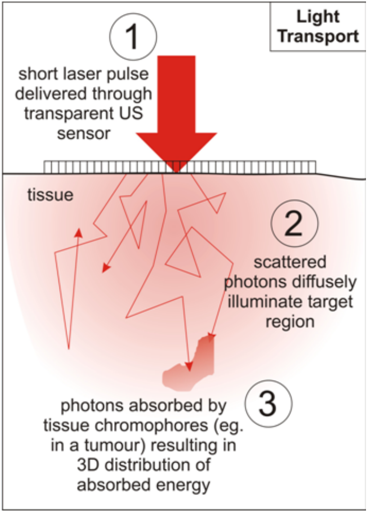
- Models of light transport
- Multispectral reconstructions
- Unknown scattering: diffusion-based inversions
- Unknown scattering: using radiative transfer equation

Outline

Optical part of the direct problem

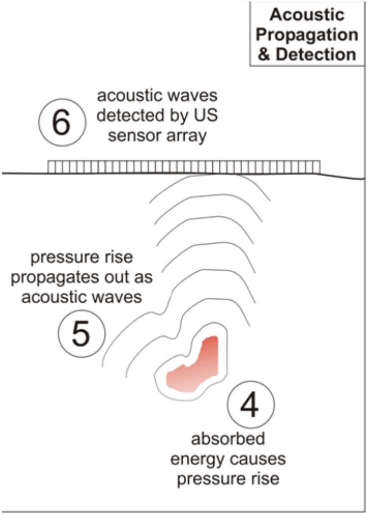
$$h(\boldsymbol{r}) = \mu_a(\boldsymbol{r})\Phi(\boldsymbol{r})$$

absorbed energy density = heat per unit volume	absorption coefficient	light fluence



Acoustic part of the direct problem

$$
\begin{aligned}
p(\boldsymbol{r})|_{t=0} &= \Gamma(\boldsymbol{r})h(\boldsymbol{r}) \\
&= \Gamma(\boldsymbol{r})\mu_{\mathrm{a}}(\boldsymbol{r})\Phi(\boldsymbol{r}) \\
&\text{Grüneisen parameter} \\
\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)p &= 0
\end{aligned}
$$



Initial value Problem

$$\begin{aligned}\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)p &= 0 \\ p\big|_{t=0} &= \Gamma\mu_{\mathrm{a}}\Phi \\ \left.\frac{\partial p}{\partial t}\right|_{t=0} &= 0\end{aligned}$$

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Boundary value Problem (t running backwards from T to 0)

$$\begin{aligned}\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right) p &= 0\\ p(\boldsymbol{r}, t)|_{t=T} &= 0\\ p(\boldsymbol{r}, t)|_{\partial\Omega} &= p^{\mathrm{obs}}(\boldsymbol{r}_s, t)\end{aligned}$$

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\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right) p &= 0 \\
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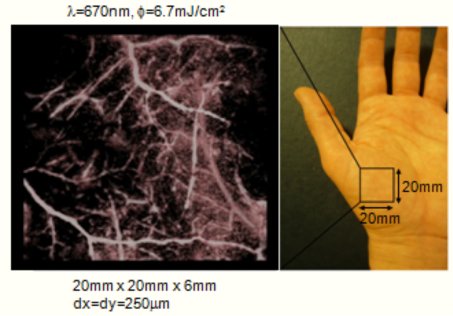

Boundary value Problem (t running backwards from T to 0)

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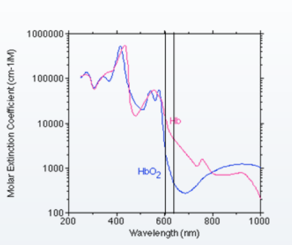
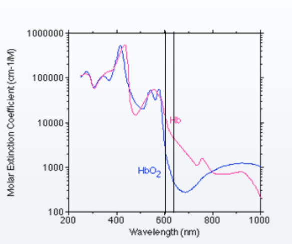
\end{aligned}
$$

- absorption at different wavelengths gives spectral images

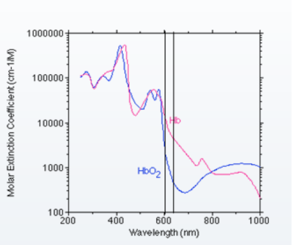


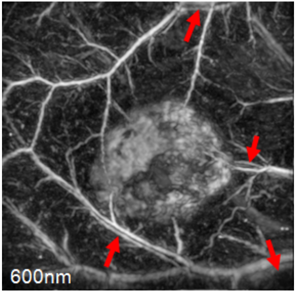
- absorption at different wavelengths gives spectral images
- but fluence is also different at different wavelengths

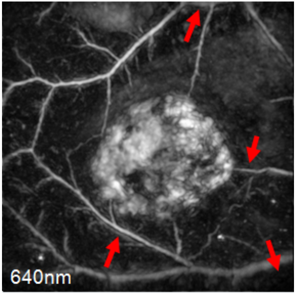


- absorption at different wavelengths gives spectral images
- but fluence is also different at different wavelengths

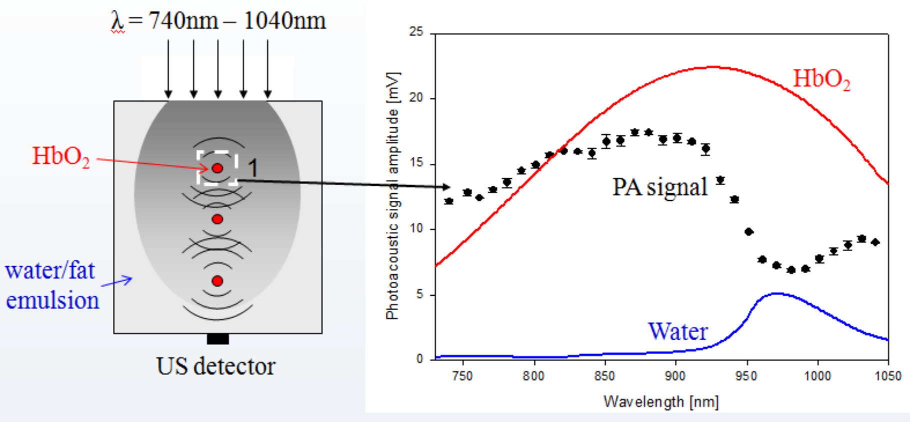
tumour type LS174T





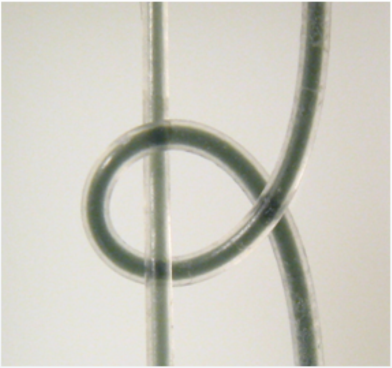


Spectral Distortion



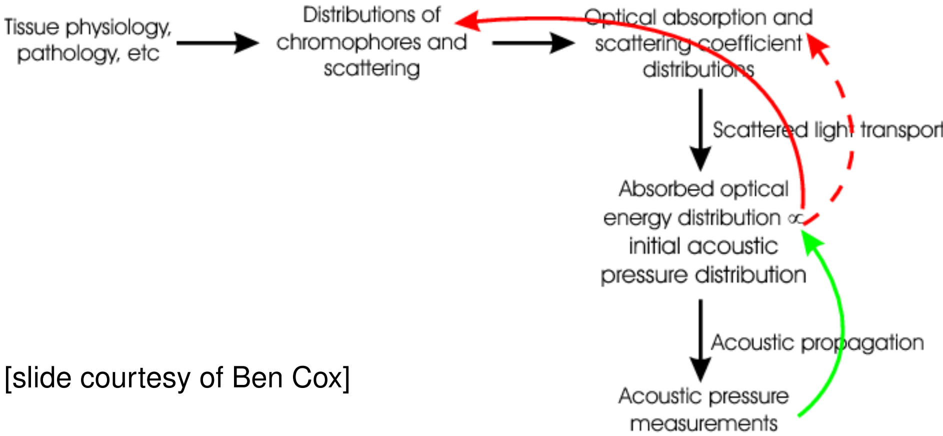
Spectrum corrupted by wavelength dependence of fluence

Structural Distortion

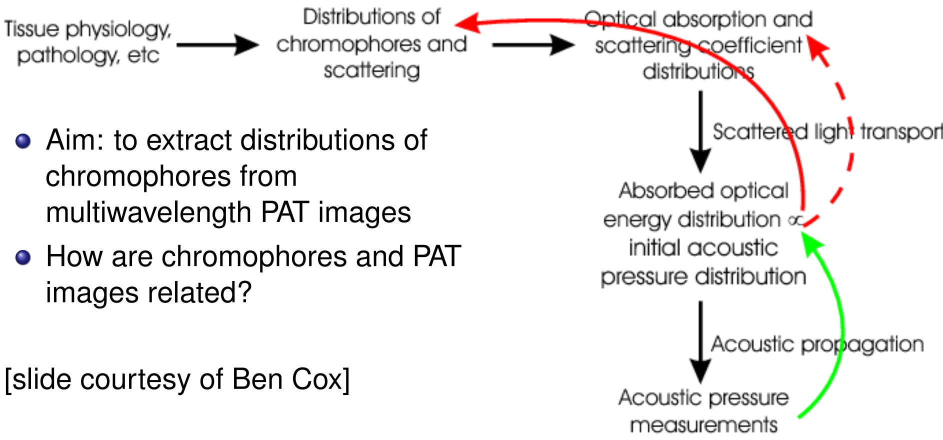


- Structural distortion due to non-uniform internal light fluence
- Structural distortion at each wavelength = spectral distortion at each point

Outline



[slide courtesy of Ben Cox]



- Aim: to extract distributions of chromophores from multiwavelength PAT images
- How are chromophores and PAT images related?

[slide courtesy of Ben Cox]

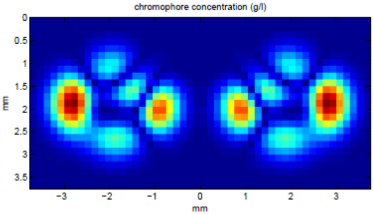
- PAT images $\propto$ absorbed energy distribution $h(\boldsymbol{r}, \lambda)$
- $p_0(\boldsymbol{r}, \lambda)$ is related to absorption coefficient $\mu_{\rm a}(\boldsymbol{r}, \lambda)$ via the fluence, $\Phi(\boldsymbol{r}, \lambda)$ and the *Grüneisen parameter*:

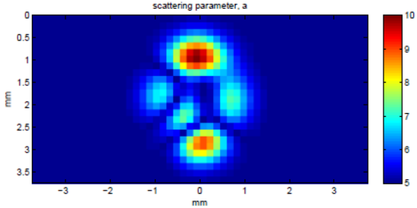
$$p_o(\boldsymbol{r}, \lambda) = \Gamma h(\boldsymbol{r}, \lambda) = \Gamma \mu_{\rm a}(\boldsymbol{r}, \lambda)\Phi(\boldsymbol{r}, \lambda)$$

- $\mu_{\rm a}$ is related to chromphores concentrations $C^{(k)}$ via specific absorption coefficients ϵ_k:

$$\mu_{\rm a}(\boldsymbol{r}, \lambda) = \sum_{k=1}^{K} \epsilon_k(\lambda) C^{(k)}(\boldsymbol{r})$$

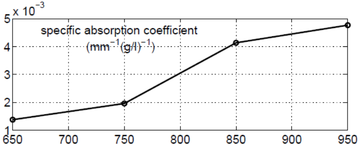
Inverse problem is non-linear but well-posed. Solve using diffusion or transport methods chromophore concentration $C(x)$ | scattering parameter $a(x)$

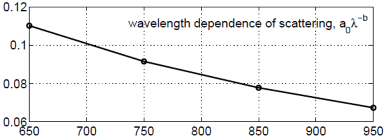




- C range: 5-15 g/l HbO_2
- $\mu_s' = a a_0 \lambda(nm)^{-b}$ mm^{-1}, $a_0 = 500$, $b = 1.3$, a range: 5-10

Wavelength Dependence

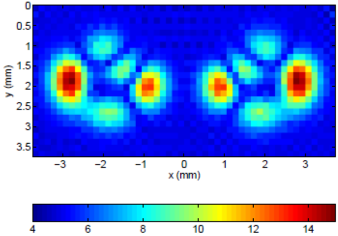


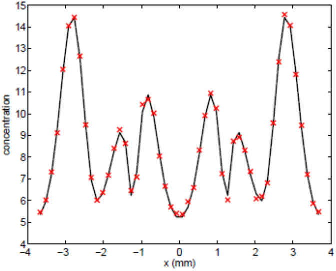


Quantitative PhotoAcoustic Tomography

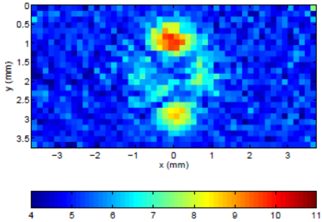
MultiSpectral QPAT reconstructions

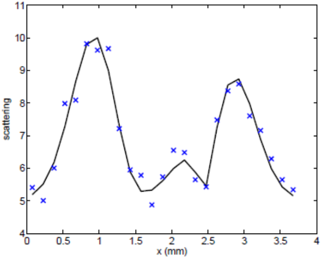
Reconstructed Concentration





Reconstructed Scattering 'a'





Find the absorption and scattering coefficients μ_a, μ_s' given the absorbed energy density image

$$h(\boldsymbol{r}) = \frac{p_0(\boldsymbol{r})}{\Gamma} = \mu_a(\boldsymbol{r})\Phi(\mu_a(\boldsymbol{r}), \mu_s(\boldsymbol{r}))$$

when the fluence Φ is unknown.

Possible strategies

- Measure internal fluence
  1. diffuse optical tomography (Yin et al. 2007)
  2. fluence-dependent chromophores (Cox, Laufer, Beard 2010)
- *Use light transport model* to model internal fluence, $\Phi(\boldsymbol{r})$

Second approach is the one used here

$$\{\hat{\mu_a}, \hat{\mu_s}\} = \arg\min_{\mu_a,\mu_s} \left[\mathcal{E} := ||h^{\text{obs}} - F(\mu_a, \mu_s')||^2 + R(\mu_a, \mu_s')\right]$$

where $F(\mu_a, \mu_s') = \mu_a\Phi((\mu_a, \mu_s')$ is the *forward model* of optical energy absorption, and R is a regularisation term.

Discretise parameters into a suitable basis

$$\mu_{\mathrm{a}}(\boldsymbol{r}) = \sum_j \mu_{\mathrm{a}j} u_j(\boldsymbol{r}), \qquad \mu_{\mathrm{s}}(\boldsymbol{r}) = \sum_j \mu_{\mathrm{s}j} u_j(\boldsymbol{r})$$

The functional gradient vectors are given by

$$\begin{aligned} g_a &= \frac{\partial \mathcal{E}}{\partial \mu_{\mathrm{a}j}} = -\sum_{m,k} (h_k^m - \mu_{\mathrm{a}k} \Phi_k^m) JA_{kj}^m + \frac{\partial R}{\partial \mu_{\mathrm{a}j}} \\ g_s &= \frac{\partial \mathcal{E}}{\partial \mu_{\mathrm{s}j}} = -\sum_{m,k} (h_k^m - \mu_{\mathrm{a}k} \Phi_k^m) JS_{kj}^m + \frac{\partial R}{\partial \mu_{\mathrm{s}j}} \end{aligned}$$

with the absorption and scattering Jacobians respectively

$$JA_{kj}^m = \Phi_k^m \delta_{kj} + \mu_{\mathrm{a}k} \frac{\partial \Phi_k^m}{\partial \mu_{\mathrm{a}j}}, \qquad JS_{kj}^m = \mu_{\mathrm{a}k} \frac{\partial \Phi_k^m}{\partial \mu_{\mathrm{s}j}}. \tag{1}$$

By combining the absorption and scattering Jacobians for every illumination into a single Jacobian matrix, $J \in \mathbb{R}^{MK \times 2K}$

$$J = \left[\begin{array}{c|c} JA^1 & JS^1 \\ \vdots & \\ JA^M & JS^M \end{array} \right],$$

the Hessian, $H \in \mathbb{R}^{2K \times 2K}$, may be approximated by $H \approx J^T J$. The update to the absorption and scattering coefficients, $[\delta\mu_{ak}, \delta\mu_{sk}]^T$, can then be calculated by a Newton step according to

$$\left[\begin{array}{c} \delta\mu_{ak} \\ \delta\mu_{sk} \end{array} \right] = -H^{-1} \left[\begin{array}{c} g_a \\ g_s \end{array} \right].$$

The radiative transfer equation is an integro-differential equation expressing the conservation of energy which takes the following time-independent form as required in PAT

$$(\hat{\boldsymbol{s}} \cdot \nabla + \mu_{\rm a}(\boldsymbol{r}) + \mu_{\rm s}(\boldsymbol{r}))\, \phi(\boldsymbol{r}, \hat{\boldsymbol{s}}) = \mu_{\rm s} \int_{S^{n-1}} \Theta(\hat{\boldsymbol{s}}, \hat{\boldsymbol{s}}') \phi(\boldsymbol{r}, \hat{\boldsymbol{s}}') d\hat{\boldsymbol{s}}' + q(\boldsymbol{r}, \hat{\boldsymbol{s}}) \tag{2}$$

where $\phi(\boldsymbol{r}, \hat{\boldsymbol{s}}, t)$ is the radiance, $\Theta(\hat{\boldsymbol{s}}, \hat{\boldsymbol{s}}')$ is the scattering phase function,
Wave effects, polarisation, radiative processes, inelastic scattering, and reactions (such as ionisation) are all neglected in this model.
By writing a variational form of equation(2) it can be discretised using the finite element method (Tarvainen2005). When the radiance ϕ, source q or phase function Θ depend strongly on the direction $\hat{\boldsymbol{s}}$ it is necessary to discretise finely in angle $\hat{\boldsymbol{s}}$, and the model can become computationally intensive.

In the Diffusion pproximation (DA), the radiance is approximated by first order spherical harmonics only ($\hat{\boldsymbol{s}} \equiv [Y_{1,-1}, Y_{1,0}, Y_{1,1}]$), giving

$$\phi(\boldsymbol{r}, \hat{\boldsymbol{s}}) \approx \frac{1}{4\pi}\Phi(\boldsymbol{r}) + \frac{3}{4\pi}\hat{\boldsymbol{s}} \cdot J(\boldsymbol{r}) \tag{3}$$

where $\Phi(\boldsymbol{r})$ and $J(\boldsymbol{r})$ are the photon density and current defined as

$$\Phi(\boldsymbol{r}) = \int_{S^{n-1}} \phi(\boldsymbol{r}, \hat{\boldsymbol{s}}) \mathrm{d}\hat{\boldsymbol{s}} \tag{4}$$

$$J(\boldsymbol{r}) = \int_{S^{n-1}} \hat{\boldsymbol{s}}\phi(\boldsymbol{r}, \hat{\boldsymbol{s}}) \mathrm{d}\hat{\boldsymbol{s}}. \tag{5}$$

Inserting the approximation (3) into equation (2) results in a second order PDE in the photon density

$$-\nabla \cdot D\nabla\Phi(\boldsymbol{r}) + \mu_{\mathrm{a}}\Phi(\boldsymbol{r}) = q_0(\boldsymbol{r}) \quad \equiv \mathcal{D}\Phi = q_0 \ , \tag{6}$$

with $D = \frac{1}{\mu_{\mathrm{a}} + (1-g)\mu_{\mathrm{s}})}$. Equation(6) and its associated frequency and time domain versions, including the Telegraph Equation, are the most commonly used in DOI.

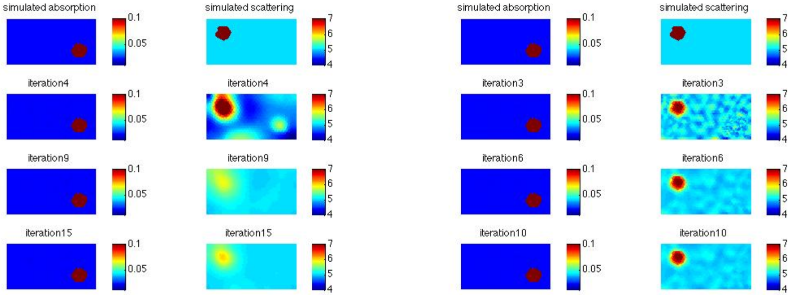
For the Radiative Transfer Equation the following equations were used to directly calculate the Jacobians *JA* and *JS* column by column

$$
\begin{aligned}
(\hat{\mathbf{s}}\cdot\nabla+\mu_{ak}+\mu_{sk})\frac{\partial\phi_k^m(\hat{\mathbf{s}})}{\partial\mu_{aj}}-\mu_{sk}\int_{S^{n-1}}\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')\frac{\partial\phi_k^m(\hat{\mathbf{s}}')}{\partial\mu_{aj}}d\hat{\mathbf{s}}' &= -\delta_{kj}\phi_k^m(\hat{\mathbf{s}}) \\
(\hat{\mathbf{s}}\cdot\nabla+\mu_{ak}+\mu_{sk})\frac{\partial\phi_k^m(\hat{\mathbf{s}})}{\partial\mu_{sj}}-\mu_{sk}\int_{S^{n-1}}\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')\frac{\partial\phi_k^m(\hat{\mathbf{s}}')}{\partial\mu_{sj}}d\hat{\mathbf{s}}' &= \\
\delta_{kj}\int_{S^{n-1}}\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')\phi_k^m(\hat{\mathbf{s}}')d\hat{\mathbf{s}}' \;-\; \delta_{kj}\phi_k^m(\hat{\mathbf{s}})
\end{aligned}
$$

For the diffusion approximation

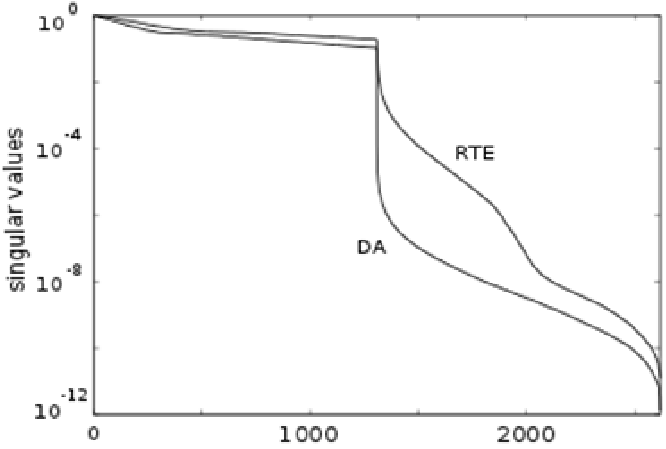
$$\begin{aligned}\left(\mu_{ak} - \nabla \cdot D_k \nabla\right) \frac{\partial \Phi_k^m}{\partial \mu_{aj}} &= -\delta_{kj} \Phi_k^m \\ \left(\mu_{ak} - \nabla \cdot D_k \nabla\right) \frac{\partial \Phi_k^m}{\partial D_j} &= \nabla \cdot \left(\delta_{kj} \nabla \Phi_k^m\right)\end{aligned}$$

$\partial \Phi_k^m / \partial \mu_{sj}$ is then obtained from $\partial \Phi_k^m / \partial D_j$ using the relation $\partial D / \partial \mu_s = -3D^2(1-g)$.



Left : iterations of QPAT using diffusion approximation. Right iterations of QPAT using RTE

SVD of Hessian reveals different information content



Outline

Summary

- Photoacoustic imaging: great potential as biomedical imaging method
- Importance of spectroscopic aspect of photoacoustics sometimes overlooked (as it is not present in thermoacoustics?)
- Much progress made in quantitative photoacoustics recently
- Linearized approaches probably not sufficient in practice
- Complete 3D problem is of large scale
- Accuracy of light models in ballistic regime (close to surface)
- Questions about Grüneisen parameter remain. (Under DA not possible to recover all of Γ, μ_{a}, D with one wavelength, but with multiple wavelengths it could be (Bal, Ren 2011, Ren, Bal 2011).)