Since interpolation problems with Blaschke products are well studied for interpolation points in the unit disc, we focus here on problems with interpolation points on the boundary. Of course, there are always finite Blaschke products B with

$$B(t_j) = w_j, \qquad j = 1, \ldots n,$$

where t_j, w_j are given points on the unit circle $\mathbb{T}$. Therefore we single out the solution of minimal degree and call it the minimal solution. As a first step towards a solution of the problem we identify three different cases of problems called rigid, flexible, and fragile, and study their different properties concerning uniqueness of solutions and stability with respect to small perturbations of the interpolation points. Then an algorithmic procedure for the solution of the boundary interpolation problem is described, leading in most cases to an exact solution in a finite number of steps in exact arithmetic.

The second part of the talk introduces phase portraits as an interesting and visually appealing tool for the representation of analytic functions. We describe how special points (such as singularities of the function or its derivative) look like in the phase portrait and give examples, how numerically generated pictures can stimulate research in complex analysis. In particular, phase portraits of Blaschke products are analyzed and we will report on progress concerning open questions in Blaschke product theory.