ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES
WITH SPECIAL EMPHASIS ON
CALABI–YAU VARIETIES AND MIRROR SYMMETRY
MARCH 19–20, 2011

ABSTRACTS

Candelas, Philip (Oxford University and Perimeter Institute)

Vacuum States of String Theory

Abstract: String theory, famously, has a great many ground states. So many, in fact, that some argue that we should seek information in the statistical properties of these vacua, or worse, argue tha t we should abandon string theory as a theory with predictive power. On the other hand, very few vacua are known that look like the observed world of particle physics. In this talk I will review this situation and show that there are realistic models at the tip of the distribution of vacua, where topological complexity is minimised.

de la Ossa, Xenia (Oxford University and Perimeter Institute)

Geometry and Connectedness of Heterotic String Compactifications with Fluxes

Abstract: I will discuss the geometry of heterotic string compactifications with fluxes. The compactifications on 6-dimensional manifolds which preserve $N = 1$ supersymmetry in 4 dimensions must be complex manifolds with vanishing first Chern class, but which are not in general Kahler (and therefore not Calabi-Yau manifolds) together with a vector bundle on the manifold which must satisfy a complicated different ial equation. The flux, which can be viewed as a torsion, is the obstruction to the manifold being Kahler. I will describe how these compactifications are connected to the more traditional compactifications on Calabi-Yau manifolds through geometric transitions like flops and conifold transitions. For instance, one can construct solutions by flopping rational curves in a Calabi-Yau manifold in such a way that the resulting manifold is no longer Kahler. Time permitting, I will discuss open problems, for example the understandin g of the the moduli space of heterotic compactifications and the relate d problem of determining the massless spectrum in the effective 4 dimen sional supersymmetric field theory. The study of these compactifications is interesting on its own right both in string theory, in order to understand more generally the degrees of freedom of these theories, and also in mathematics. For instance, the connectedness between the solutions is related to problems in mathematics like the conjecture by Mile Reid that complex manifolds with trivial canonical bundle are all connected through geometric transitions.

Garavuso, Richard (University of Alberta)

Hori-Vafa mirror periods, Picard-Fuchs equations, and Berglund-Hübsch-Krawitz duality

Abstract: This paper discusses the overlap of the Hori-Vafa formulation of mirror symmetry with some other constructions. We focus on compact Calabi-Yau hypersurfaces $\mathcal{M}_G = \{G = 0\}$ in weighted complex projective spaces. The Hori-Vafa formalism relates a family

$$\left\{ \mathcal{M}_G \in \mathbf{WCP}^{m-1}_{Q_1,\ldots,Q_m}[s] \,\middle|\, \sum_{i=1}^{m} Q_i = s \right\}$$

of such hypersurfaces to a single Landau-Ginzburg mirror theory. A technique suggested by Hori and Vafa allows the Picard-Fuchs equations satisfied by the corresponding mirror periods to be determined. Some examples in which the variety $\mathcal{M}_G$ is crepantly resolved are considered. The resulting Picard-Fuchs equations

agree with those found elsewhere working in the Batyrev-Borisov framework. When G is an invertible non-degenerate quasihomogeneous polynomial, the Chiodo-Ruan geometrical interpretation of Berglund-Hübsch-Krawitz duality can be used to associate a particular complex structure for $\mathcal{M}_G$ with a particular Kähler structure for the mirror $\widetilde{\mathcal{M}}_G$. We make this association for such G when the ambient space of $\mathcal{M}_G$ is $\mathbf{CP}^2$, $\mathbf{CP}^3$, and $\mathbf{CP}^4$. Finally, we probe some of the resulting mirror Kähler structures by determining corresponding Picard-Fuchs equations.

Reference: C. F. Doran and R. S. Garavuso, *Hori-Vafa mirror periods, Picard-Fuchs equations, and Berglund-Hübsch-Krawitz duality*, to appear in JHEP, http://arxiv.org/abs/1109.1686; arXiv:1109.1686.

Gualtieri, Marco, University of Toronto

Deformations of Generalized Kahler manifolds

Abstract: Underlying any generalized Kahler manifold is a holomorphic Poisson variety whose Poisson structure "facto rizes" in a certain sense into a product of holomorphic Di rac structures. We will describe some aspects of the defor mation theory of these structures and compare it to the de formation theory of Calabi-Yau varieties.

Haessig, Doug (University of Rochester)

Unit roots everywhere! (aka. Variation of the unit root of a family of unit root L-functions)

Abstract: Using pioneering work of Dwork as a template, JD Yu recently looked at the variation of the zeta function through a certain family of Calabi-Yau varieties. He showed that at each fiber, the unique unit root of the zeta function may be obtained through specialization of a single p-adic analytic function coming from the horizontal section of the Gauss-Manin connection on the family. The theme of this talk is similar. However, we will look at the variation of the unit root coming from a family of L-functions defined by unit roots.This is work in progress with Steven Sperber.

Kerr, Matt (Washington University of St. Louis)

Special values of automorphic cohomology classes

Abstract: We propose a definition of cuspidal automorphic cohomology for a class of period domains related to Calabi-Yau threefolds, and discuss several related arithmetic structures which can be placed on these cohomology groups. This is joint work with M. Green and P. Griffiths.

Long, Ling (Iowa State University)

Some automorphic results on Galois representations and applications

Abstract: In this talk, we consider Galois representations attached to noncongruence cuspforms constructed by Tony Scholl which are natural generalizations of Deligne's ℓ-adic representations attached to classical Hecke eigenforms. Scholl representations constitute a large class of Galois representations which include Galois representations arising from the Tate modules of smooth projective indecomposable curves over number fields. We will prove under special circumstances that these Galois representations are automorphic in the sense that their L-functions agree with the L-functions of automorphic forms on reductive groups and then give some applications of such automorphic results.

Moraru, Ruxandra (University of Waterloo)

Compact moduli spaces of stable bundles on Kodaira surfaces

Abstract: In this talk, I will examine the geometry of moduli spaces of stable bundles on Kodaira surfaces, which are non-Kaehler compact surfaces that can be realised as torus fibrations over elliptic curves. These moduli spaces are interesting examples of holomorphic symplectic manifolds whose geometry is similar to the geometry of Mukai's moduli spaces on K3 and abelian surfaces. In particular, for certain choices of

rank and Chern classes, the moduli spaces are themselves Kodaira surfaces.

This is joint work with Marian Aprodu and Matei Toma.

Pearlstein, Greg (Michigan State University)

Normal functions and the Hodge conjecture

I will summarize recent work on the Hodge conjecture using normal functions, and the relationship with archimedean heights.

Rayan, Steven (University of Toronto)

Betti numbers of twisted Higgs moduli spaces at genus 0, through quivers

Abstract: We will apply the Morse-theoretic approach of Hitchin and Gothen for computing Betti numbers of Higgs bundle moduli spaces in low rank to moduli spaces of twisted Higgs bundles on $\mathbf{P}^1$. We highlight similarities and differences between usual Higgs moduli spaces in positive genus and the twisted ones in genus 0, with respect to the S^1 Morse theory. Certain difficulties encountered when extending the method to Higgs moduli spaces of rank larger than 3 are surmountable in the very combinatorial twisted Higgs case on $\mathbf{P}^1$. These calculations require us to study holomorphic chains on $\mathbf{P}^1$. The approach we take involves quivers, and we develop a dictionary between holomorphic chains and certain classes of quivers. In particular, our calculations show that a twisted ADHM formula of Mozgovoy (after Chuang, Diaconescu, and Pan) gives exactly the Poincare polynomials of twisted Higgs moduli spaces, at least for ranks 1 through 5. Our calculations combined with the ADHM data suggest some conjectures about Betti numbers for higher rank moduli spaces.

Zhong, Changlong (University of Ottawa)

Comparison Dualizing Complexes

Abstract: In this talk I will introduce four dualizing complexes defined by M. Spiess, T. Moser, S. Bloch (duality proved by T. Geisser) and K. Sato, and compare them in the derived category. We show that Bloch's complex is quasi-isomorphic with all the three in the situation when they are properly defined (and assuming some well-known conjectures).