THE CORONA PROBLEM: CONNECTIONS BETWEEN OPERATOR THEORY, FUNCTION THEORY AND GEOMETRY

Schedule:

Monday:
9:30-10:30 Talk: Ron G. Douglas
10:30-11:00 Coffee Break
11:00-12:00 Talk: Steven G. Krantz
12:00-2:00 Lunch
2:00-3:00 Talk: Eric T. Sawyer
3:00-3:30 Coffee Break
3:30-4:00 Talk: Mishko Mitkovski
4pm- Open

Tuesday:
9:30-10:30 Talk: Sergei Treil
10:30-11:00 Coffee Break
11:00-12:00 Talk: Tavan Trent
12:00-2:00 Lunch
2:00-3:00 Talk: Alexander Brudnyi
3:00-3:30 Coffee Break
3:30-4:00 Talk: Kelly Bickel
4pm- Open

Wednesday:
9:00-2:00 Free
2:00-3:00 Open Problem Session
3:00-3:30 Coffee Break
3:30-4:30 Back2Fields Colloquium: Brett D. Wick

Thursday:
9:30-10:30 Talk: Nikolai Nikolskii
10:30-11:00 Coffee Break
11:00-12:00 Talk: Amol Sasane
12:00-2:00 Lunch
2:00-3:00 Talk: Greg Knese
3:00-3:30 Coffee Break
3:30-4:00 Talk: Ryan Hamilton
4pm- Open

Friday:
9:30-10:30 Talk: Brett D. Wick
10:30-11:00 Coffee Break

11:00-12:00 Talk: Richard Rochberg
12:00-2:00 Lunch
2:00-3:00 Talk: Ken Davidson
3:00-3:30 Coffee Break
3:30-4:00 Talk: Debendra Banjade

Titles and Abstracts:

Speaker: Debendra Banjade
Title: Wolff's Corona Problem in the Multiplier Algebra of the Dirichlet Space in the unit disk

Abstract: Let $\{f_j\}_{j=1}^{\infty} \subset \mathcal{M}(\mathcal{D})$, where $\mathcal{M}(\mathcal{D})$ is the multiplier algebra of the Dirichlet space $\mathcal{D}$. We let $F(z) = (f_1(z), f_2(z), \ldots)$, $z \in D$. In this talk we will give a sufficient condition for the existence of $G(z) = (g_1(z), g_2(z), \ldots)$, $\{f_j\}_{j=1}^{\infty} \subset \mathcal{M}(\mathcal{D})$ such that $H(z) = F(z)G(z) \forall z \in D$. In other words, we are generalizing Wolff's theorem from $H^{\infty}(D)$ to $\mathcal{M}(\mathcal{D})$.

Speaker: Kelly Bickel
Title: Inner Functions and associated Hilbert Spaces on the Bidisk

Abstract:Although is well known that operator-valued inner functions on the bidisk admit decompositions in terms of two reproducing kernels, called Agler Decompositions, the structure of such decompositions has remained fairly mysterious.

In this talk, We discuss an elementary proof of the existence of such Agler decompositions, which constructive and yields cannonical decompositions. These cannonical decompositions can be used to characterize all Agler decompositions and provide a uniqueness criterion. We then analyze the structure and regularity of the cannonical decompositions and show how the properties can be extended to general Agler decompositions. This is joint work with Greg Knese.

Speaker: Alex Brudnyi
Title: Stein-like Theory for Banach-valued Holomorphic Functions on the Maximal Ideal space of H^{∞} and the Operator Corona Problem of Sz.-Nagy

Abstract: In the talk I describe my results presented in two preprints arXiv:1103.2347 and arXiv:1103.5237 concerning Banach-valued holomorphic functions defined on open subsets of the maximal ideal space $M(H^{\infty})$ of the Banach algebra H^{∞} of bounded holomorphic functions on the unit disk $\mathbb{D} \subset \mathbb{C}$ with pointwise multiplication and supremum norm. These include vanishing cohomology for sheaves of germs of such functions, corona theorems for algebras $H^{\infty}_{\mathrm{comp}}(A)$ of holomorphic functions on $\mathbb{D}$ with relatively compact images in commutative unital complex Banach algebras A, the Oka principle for holomorphic Banach vector bundles on $M(H^{\infty})$ and its applications to the Sz.-Nagy operator corona problem.

Speaker: Ken Davidson
Title: Classification of Universal Operator Algebras associated to Varieties

Abstract: I will discuss the universal operator algebra of a commuting row contraction that satisfies relations associated to the ideal of functions vanishing on a variety. The goal is to classify these algebras up to isomorphism. We show that two such algebras are completely isometrically isomorphic if and only if there is a conformal automorphism of the ball in $\mathbb{C}^n$ carring one variety (intersected with the ball) onto the other. In the case of a homogeneous ideal, we show that the two algebras are algebraically isomorphic if and only if the two varieties intersected with the ball are biholomorphic. (This relies in part on a recent result of Michael Hartz.) In the non-homogeneous case, isomorphism implies the the varieties are biholomorphic; but counterexamples show that the converse fails. (This is joint work with Chris Ramsey and Orr Shalit.)

Speaker: Ron Douglas
Title: The Corona Problem and Operator Theory

Abstract: One fascinating aspect of the Corona Problem is that it sets at the nexus of so many areas of analysis and geometry. Although motivated by an abstract question in function algebras, its initial solution for the unit disk by Carleson involved techniques from harmonic analysis and function theory which have had far reaching consequences in large parts of analysis, in many cases raising questions with solutions that provide deep insights. But that is not all. In particular, the corona problem can be restated in terms of problems in operator theory, particularly on various Hilbert spaces of holomorphic functions.

In this talk I will try to outline some of the developments surrounding the original efforts to solve the problem as well as connections of the problem with operator theory. These ideas are of particular importance in efforts to frame analogues of the corona problem in the several variable case.

Speaker: Ryan Hamilton
Title: The Toeplitz Corona Problem and a Distance Formula

Abstract: The Toeplitz corona problem for H^∞, solved by Arveson in the 70s, replaces the classical corona condition with a stronger operator inequality. The solution of optimal norm was discovered a few years later through a simple proof of Schubert. In this talk, we will examine the Toeplitz corona problem in a general setting, and describe some results along the lines of recent papers of Douglas-Sarkar, Trent-Wick and Raghupathi-Wick. This approach will rely on some deep results from dual algebra theory from the 1980s. Much of this talk is based on joint work with Raghupathi (USNA).

Speaker: Greg Knese
Title: Inner functions on the bidisk and associated Hilbert spaces

Abstract: Matrix valued inner functions on the bidisk have a number of natural subspaces of the Hardy space on the torus associated to them. We study their relationship to Agler decompositions, regularity up to the boundary, and restriction maps into one variable spaces. We give a complete description of the important spaces associated to matrix rational inner functions. The dimension of some of these spaces can be computed in a straightforward way, and this ends up having an application to the study of three variable rational inner

functions. This is joint work with Kelly Bickel.

Speaker: Steven Krantz
Title: The Corona Problem In Several Complex Variables

Abstract: Our knowledge of the corona problem in several complex variables is sketchy. In this talk we showcase a couple of key papers that help to chart the way, and that indicate some possible directions of investigation. Necessary background in the function theory of several complex variables is minimal.

Speaker: Mishko Mitkovski
Title: Compactness of operators on Bergman type spaces

Abstract: I will present a characterization of the compact operators on a class of Bergman type spaces in the following form: An operator is compact if and only if it belongs in the corresponding Toeplitz algebra and its Berezin transform vanishes at the boundary. In addition, I will show how our approach gives a precise estimate of the essential norm of all operators in the Toeplitz algebra of this spaces. The crucial property that allows us to do this is the possibility of localization of the reproducing kernels in this spaces. This talk is based on joint work with B. Wick.

Speaker: Nikolai Nikolski
Title: Numerically Effective Corona Problems

Abstract: The effective corona problem in a commutative Banach algebra is to find a "critical" value δ_n, $0 \le \delta_n \le 1$ (the smallest one), guaranteeing the solvability of n-term Bezout equations $\sum_1^n g_k f_k = 1$ for δ-flat data $\delta_n < \delta \le |f(x)| \le \|f\| \le 1$ with an estimate of a solution g depending on δ only. Several examples are considered: (1) Convolution measure algebras, (2) Quotient H^∞ algebras, (3) Fourier-Muckenhoupt multipliers. If the time permits, applications to effective matrix inversions can be discussed.

Speaker: Richard Rochberg
Title: Truncated Hankel forms, Hankel forms on Model Spaces

Abstract: Truncated Toeplitz operators (TTOs) are Toeplitz operators compressed to a Model Space K, the orthogonal complement of an invariant subspace of the Hardy space. TTOs have been studied actively since Sarason's 2007 paper focused attention on them. I will describe some of the recent progress and mention open questions. I will introduce the corresponding theory of Hankel forms on K. Each K has a conjugation operator, a conjugate linear isometric involution. The conjugation creates a very intimate link between the Hankel and Toeplitz theories. That link suggests a unified viewpoint which leads to new results and new open questions for both the Toeplitz and Hankel theories. I will give several examples.

Speaker: Amol Sasane
Title: The corona and completion problem for an algebra of measures used in control theory

Abstract: We consider a Banach algebra, arising in control theory, consisting of the Laplace transforms of complex Borel measures on the real line, with support in the half line of non-negative real numbers, which do not have a singular nonatomic part. We show that the corona theorem holds, and left invertible matrices with entries from this Banach algebra can be completed to invertible ones. We will also discuss consequences of these properties in the stabilization problem in control theory.

Speaker: Eric Sawyer
Title: Corona theorems for the Drury-Arveson space and other Besov-Sobolev spaces of holomorphic functions on the ball

Abstract: We discuss the ideas behind the absence of a corona in the multiplier algebra of the Drury-Arveson space. Included are the Toeplitz corona theorem, the Koszul complex, Charpentier's solution operators to d-bar equations, and the interplay with complex tangential vector fields. These ideas extend to other Besov-Sobolev spaces of holomorphic functions on the ball having varying degrees of smoothness, as well as to vector-valued settings.

Speaker: Tavan Trent
Title: Remarks on Corona Problems

Abstract: We outline a Hilbert space approach to corona problems for the algebra of multipliers on reproducing kernel Hilbert spaces. Then we show the relevance of complete NP kernels to corona problems. Next, we show how a duality argument applies in the general case to reduce the classical corona problem on the polydisk to an equivalent estimate.

Speaker: Sergei Treil
Title: Corona, ideals, and the completion problem: a geometric approach

Abstract: A surprising lemma due to N. Nikolski states that a bounded analytic matrix or operator-valued function with a bounded left inverse admits a bounded analytic left inverse if and only if there exists a bounded analytic projection onto its range. This lemma and its modifications give us a new point of point of view on the corona and related problems. It trivializes the solution of the completion problem in the disc, and might give a useful insight for more general domains. For the operator corona problem (the problem of analytic left invertibility) it gives, in particular, a complete solution in the case of finite codimension. And it also gives a new and presumably sharp result in the problem of ideals of H^∞. In my talk I'll explain the connection between these problems and analytic projections, and present some open problems.

Speaker: Brett D. Wick
Title: BMO Estimates for the Corona Problem on the Ball

Abstract: The Corona problem for bounded analytic functions in several complex variables is a well-known open question. In the late 70s it was shown by Varopoulos that there are estimates for solutions to this problem, but in BMO. In this talk we will give another proof of this result that provides an added extension to the case of vector-valued functions. This

talk is based on joint work with S. Costea and E. Sawyer.