Title and abstract for my lectures in Toronto:

Title: Locally compact quantum groups

Abstract: To any locally compact group G, one can associate two C*-algebras. First there is the algebra $C_0(G)$ of complex functions on G tending to 0 at infinity. Next, we have the reduced C*-algebra $C_r^*(G)$. There is a natural duality between the two in such a way that the product on one component induces a coproduct on the other one. The coproduct Δ on $C_0(G)$ is given by the formula $\Delta(f)(p,q) = f(pq)$ whenever $p, q \in G$. The coproduct Δ on $C_r^*(G)$ is characterized by $\Delta(\lambda_p) = \lambda_p \otimes \lambda_p$ where $p \mapsto \lambda_p$ is the canonical imbedding of G in the multiplier algebra of $C_r^*(G)$.

The above duality generalizes the Pontryagin duality for locally compact abelian groups to the non-abelian case.

In the theory of locally compact quantum groups, the idea is to 'quantize' the above system. The abelian C*-algebra $C_0(G)$ is replaced by any C*-algebra A and the coproduct Δ on $C_0(G)$, induced by the product in G, is replaced by any coproduct Δ on the C*-algebra A. Further assumptions on the pair (A, Δ) are necessary for it to be called a locally compact quantum group. Then the dual can be constructed and it is again a locally compact quantum group.

In the general theory, unfortunately, the existence of the quantum analogues of the Haar measures, the Haar weights on the pair (A, Δ), has to be assumed. On the other hand, as it turns out, these Haar weights are unique, if they exist. And moreover, in examples, there are most of the time obvious candidates for which it is not difficult to prove that they satisfy the requirements.

There is also a formulation of the theory in the setting of von Neumann algebras. This is not so natural, from a philosophical point of view, but on the other hand, it seems to allow an easier treatment. And as the two approaches are completely equivalent and in the end yield the same objects, we will follow the more easy von Neumann algebraic track to develop the theory. Still we will explain how this can be used to understand the C*-approach as well.

Content of the five lectures:

1. The Haar weights on a locally compact quantum group
2. The antipode of a locally compact quantum group
3. The main results about locally compact quantum groups
4. The dual of a locally compact quantum group
5. Special cases, examples and generalizations

In the first lecture, we will need to review the basics of the theory of lower semi-continuous weights on C*-algebras and normal weights on von Neumann algebras, in relation with various aspects of the Tomita-Takesaki theory. In the middle three lectures, we will develop the theory. And in the last lecture, if time permits, we will also say something about the various directions of recent developments.

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