# A little bit different quantum-state tomography

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# Standard QSE

*experiment*

- well-known measurement: $\Pi_j$
- probabilities: $p_j = \mathrm{Tr}(\rho \Pi_j)$
- data: $f_j$

*reconstruction*

- choice of reconstruction space: $S = \sum_j^d |j\rangle\langle j|$
- data fitting: $\rho_{\mathrm{est}}$ maximizing a given cost function

# Standard QSE ...

*some known issues*

- knowledge of the measurement required
- result may strongly depend on the reconstruction space
  - classical/non-classical
  - separable/entangled
- imperfect knowledge of the apparatus
  - bias
  - reconstruction artifacts
  - badly conditioned schemes and/or large recon. spaces: reconstruction breaks down

# Data pattern tomography

*key features*

- prior knowledge of the apparatus is not required
- estimator is a mixture of experimentally feasible probe states
- reconstruction space is spanned by the probe states
- *field of view* is determined by the quantum resources used in the experiment

# Procedure

- probe states $\sigma_k$ measured
- data patterns $f_j^k$ recorded
- unknown state $\rho$ measured and data $f_j$ recorded

- best fit of $f_j$ in terms of $f_j^k$ found: $f_{j,\mathrm{est}} = \sum_k x_k f_j^k$
- estimator: $\rho_{\mathrm{est}} = \sum_k x_k \sigma_k$

- quantum theory enters the procedure through positivity constraints

# Procedure ...

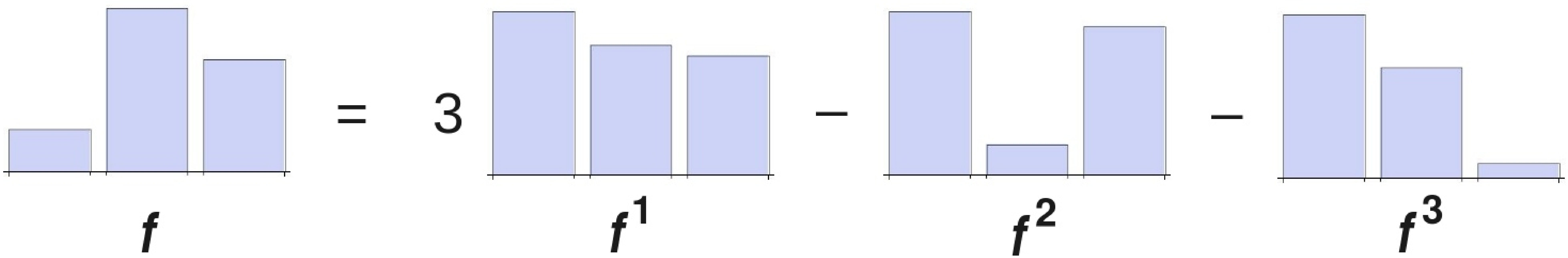
*data pattern tomography*

- find $\boldsymbol{x}$ minimizing $\mathrm{dist}\left(\boldsymbol{f}, \sum_k x_k \boldsymbol{f}^k\right)$
- subject to $\rho_{\mathrm{est}} = \sum_k x_k \sigma_k$ being non-negative $\rho_{\mathrm{est}} \geq 0$

*numerical implementation*

- least square fit
- convex programming

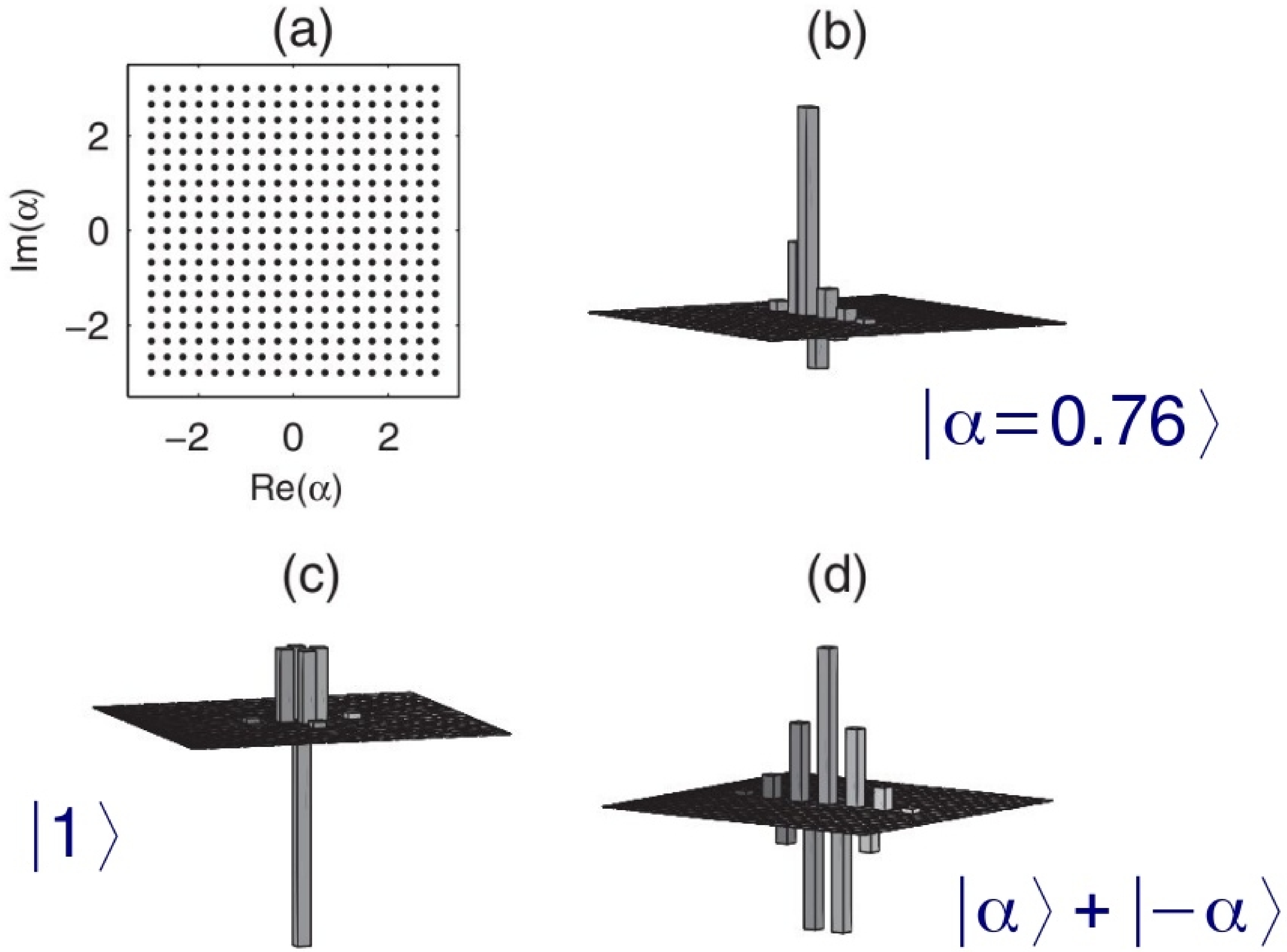
# Example




reconstruction: $\rho = 3\sigma_1 - \sigma_2 - \sigma_3$

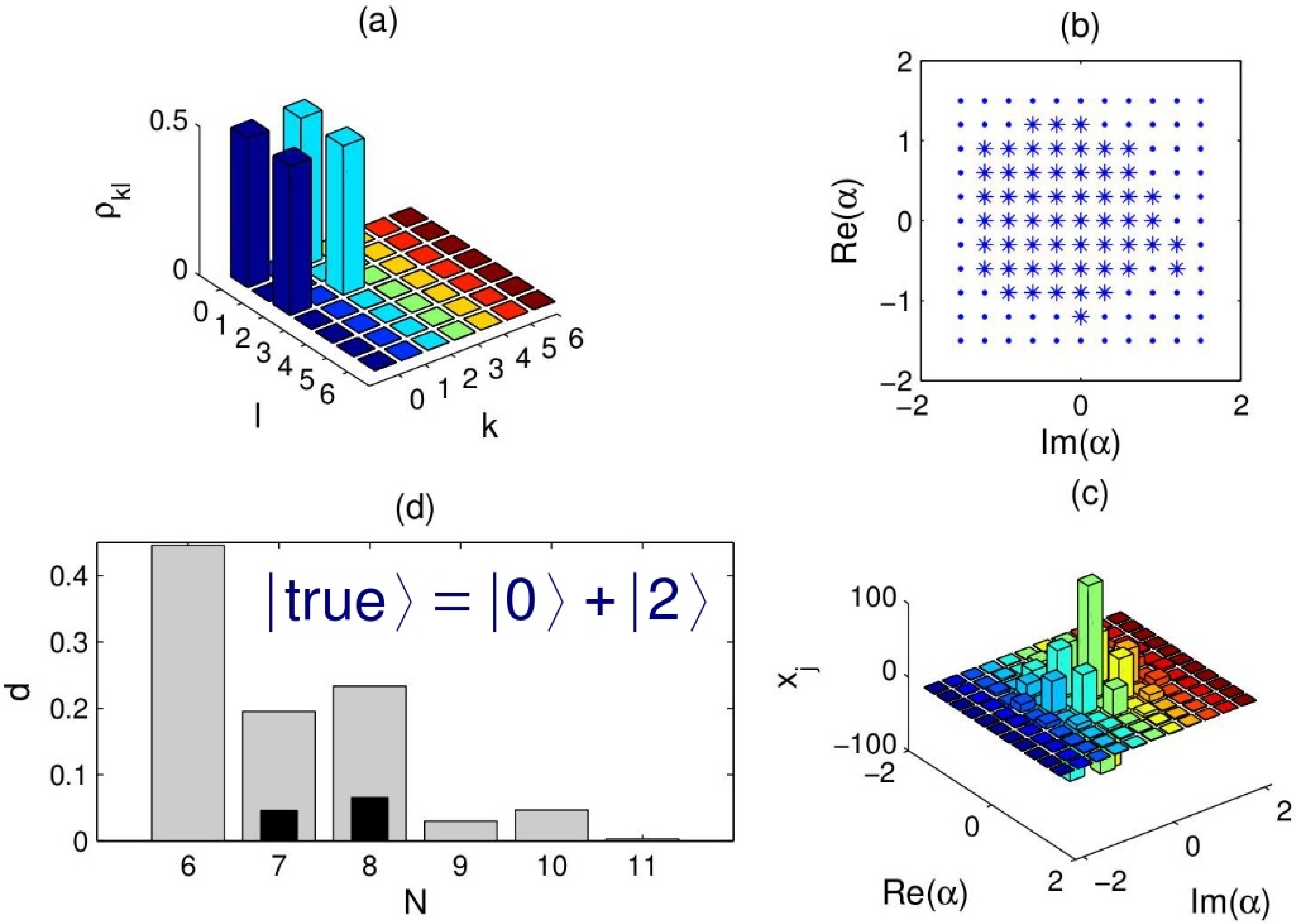
# State representation

- coherent-state representation



# State representation ...

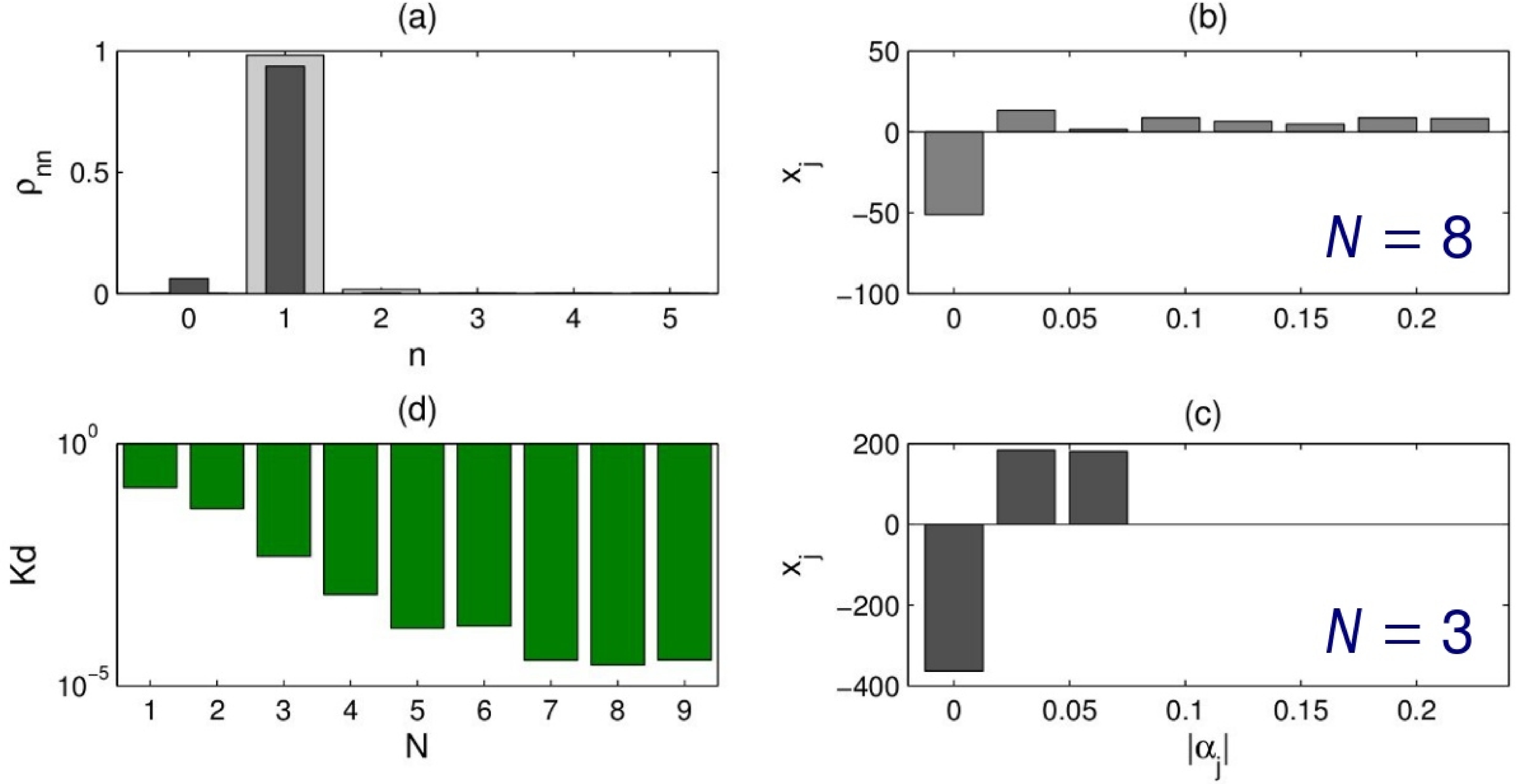
- coherent states and thermal state(s)



# Example 1: photon counting

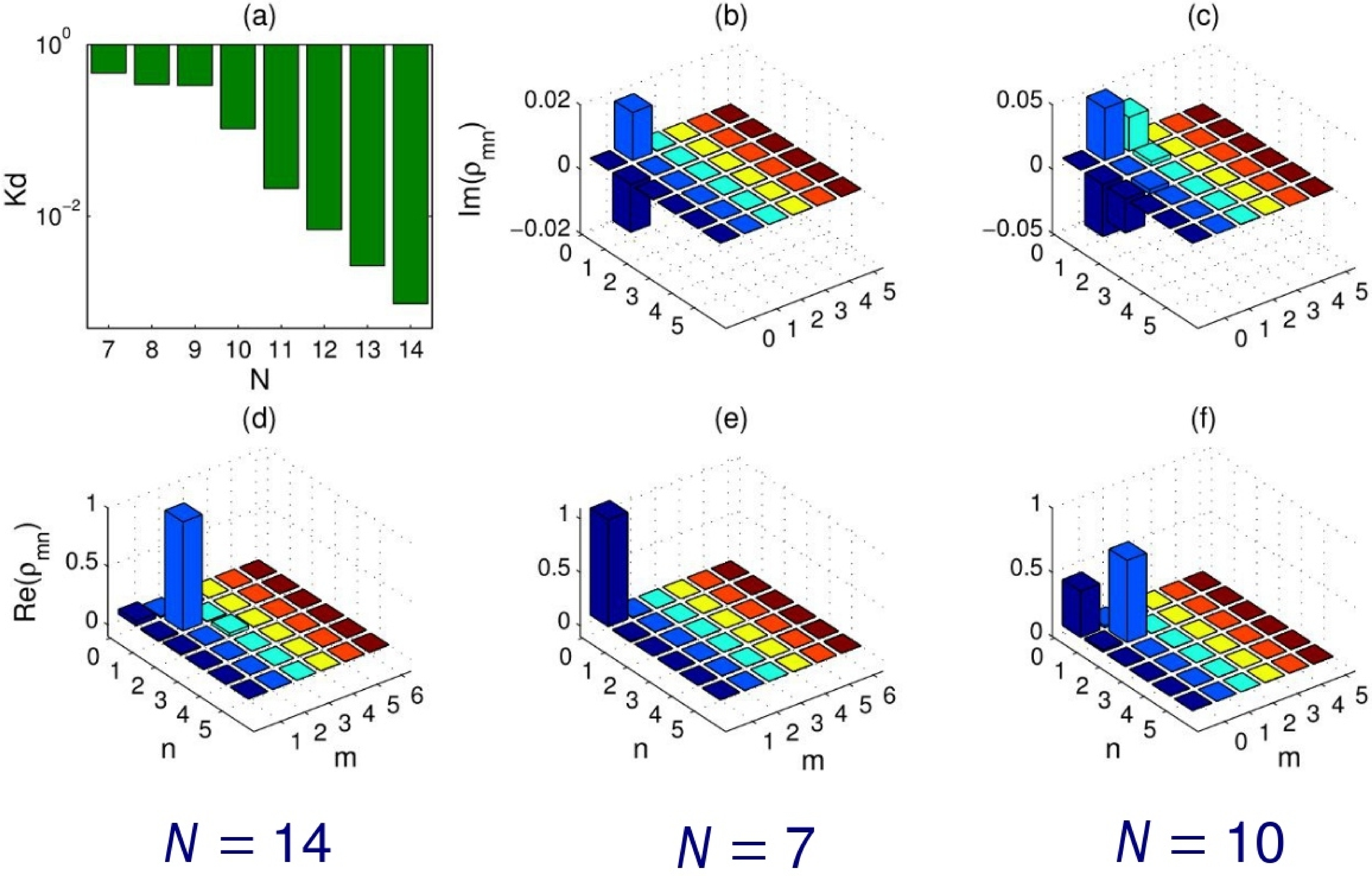
- phase-averaged coherent probe states

$$p_j = \sum_n (1-\eta_j)^n p_n$$



# Example 2: homodyne tomography

- *NxN* grid of coherent probe states



# Conclusions

- Data pattern tomography seems to be a promising alternative to standard QSE methods.
- No prior calibration of the measurement apparatus is necessary.
- In fact tomography with an unknown measuring apparatus is possible.
- Result is based on quantum resources actually used in the experiment.
- *Field of view* is uniquely defined by the measurement.

# IC of CV measurements

In a finite subspace

$$S = \sum_{n}^{dim} |n\rangle\langle n|$$

a von Neumann measurement

$$\sum_{x}^{\infty} |x\rangle\langle x| = \hat{1}, \quad \langle x|x'\rangle = \delta_{xx'}$$

becomes

$$\sum_{x} \Pi_x \equiv \sum_{x} S|x\rangle\langle x| S = \hat{1}_S$$

Notice that (in general)

$$[\Pi_x, \Pi_{x'}] \neq 0$$

# Example: homodyne detection

- quadrature measurements

$$|x_\theta\rangle\langle x_\theta|$$

- Fock subspace

$$S = \sum_{n=0}^{d-1} |n\rangle\langle n|$$

- probabilities

$$p_{(x,\theta)} = \sum_{kl} \rho_{kl} \, H_k(x) \, H_l(x) e^{-x^2} e^{i(k-l)\theta}$$

- how many independent observations generated?

# Example ...

- size of POVM

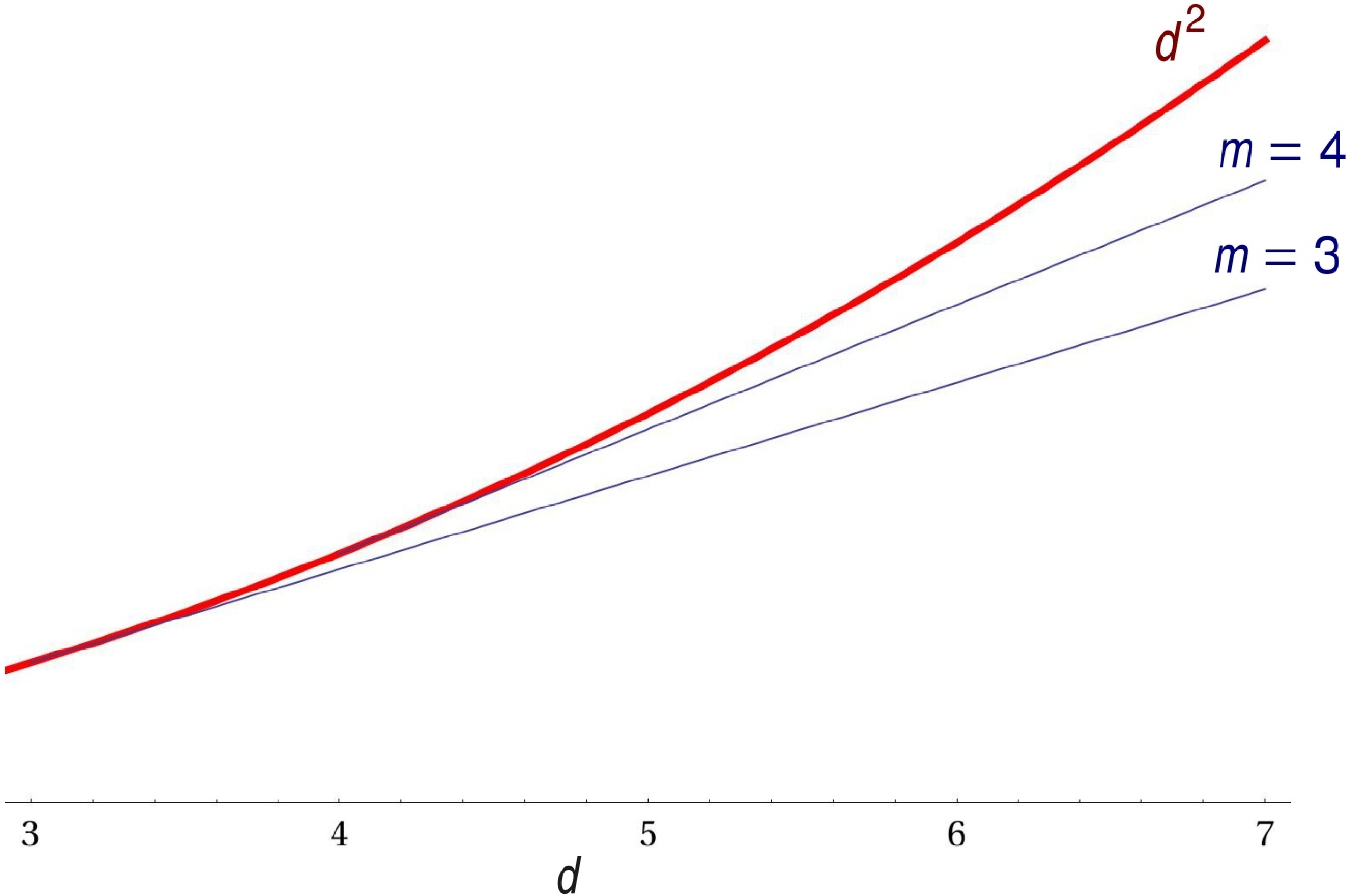
  $m$ quadratures

$$N = \begin{cases} m(2d-m), & m < d \\ d^{2}, & m \geq d \end{cases}$$

| $d$ | $m=1$ | $m=2$ | $m=3$ |
|---|---|---|---|
| 2 | 3 | **4** | **4** |
| 3 | 5 | 8 | **9** |
| 4 | 7 | 12 | 15 |
| 5 | 9 | 16 | 21 |

qutrit!

# Plot



# Conclusions

- Any CV measurement is IC *somewhere*.
- In a finite subspace of interest a POVM is induced that may or may not be IC.
- A single von Neumann measurement can be used to characterize quantum systems of arbitrary dimension.
- In the case of homodyne detection, the number of independent POVM elements increases linearly with the dimension of the reconstruction subspace.