Uniqueness theorem for certain 1-dimensional NCCW with non-trivial K_1-group

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- $\phi^* \cong \psi^*$ at the level of **the invariant** then
- ϕ and ψ are **equivalent** in a suitable sense
- often one of the maps $\phi : A \to B$ is a *-homomorphism of **special form**
  often called "standard form" or "diagonal form"

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- $r = 0$ means unital *-homomorphism

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- Application: one-step Bratteli diagram

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- Note: $\sum_{i=1}^{p} m_{ij} k_j \leq n_i$, for $1 \leq i \leq s$,
  with equality if the map is unital.

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- $$\phi(\lambda, \mu) = \left[\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} \right]$$

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- $$A \cong B$$

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- stably isomorphic algebras
- (Robert) extended this for 1-dim NCCW with K_1-group trivial, using Cuntz semigroup

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- The index set I can be chosen to be pairs (F, ϵ) with order: $(F, \epsilon) \prec (F', \epsilon')$ if $F \subseteq F'$ and $\epsilon' \leq \epsilon$.

The building blocks

- "point-line" alg. can be described by two one step Bratteli diagram: given f.d. C and D and two *-homo (left and right) from C to D with prescribed Bratteli diagrams so that C is mapped inj. into D, then alg. A is bdd. cont. from real line to D, convg. at inf. to the left/right images of *-homo.

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- Algebras A are: extensions of $M_n(\mathbb{C}) \otimes C_0(0,1)$ by a finite dimensional algebra $M_q(\mathbb{C})$

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- alternative description, assuming $n = (a+2)q$, for some a positive integer:
$A = \{f \in C([0,1], M_n(\mathbb{C}) : f(0) = diag(M_q, \ldots M_q, 0), f(1) = diag(M_q, \ldots M_q)\}$

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- Note: the size of 0 in $f(0)$ is $2q$. It follows that A is non-unital, stably projectionless.

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  if A satisfies UCT and B is σ-unital.

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- there is $\Phi : A \rightarrow B$ iso. that induces the given maps at the invariant level

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- (iv) there exists a $\delta > 0$ such that ϕ and ψ are within δ on traces
  i.e. $|\tau(\phi(f) - \psi(f))| < \delta, \tau(\phi(f)) > \delta, \tau(\psi(f)) > \delta, \tau \in T(B)$

$$f \in H$$

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  i.e. $|\tau(\phi(f) - \psi(f))| < \delta, \tau(\phi(f)) > \delta, \tau(\psi(f)) > \delta, \tau \in T(B)$
  $$f \in H$$
- then the maps ψ and ϕ are approx. unit. equivalent

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KK-groups

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- Claim: $K^0(A) \cong \mathbb{Z} \oplus K_1(A)$

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