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# Solid-state quantum communications and quantum computation based on single quantum-dot spin in optical microcavities

Chengyong Hu and John G. Rarity

Electrical & Electronic Engineering, University of Bristol  
United Kingdom

[chengyong.hu@bristol.ac.uk](mailto:chengyong.hu@bristol.ac.uk)



## Background: Semiconductor quantum dots for QIP

- Artificial atoms: QD, NV, superconductor...
- QDs are scalable, compatible with semiconductor technology
- Flying qubit - photon
  - ✓ Single photon sources
    - Electrically /optically driven
    - Indistinguishable
    - Cavity-QED enhanced
  - ✓ Entangled photon-pair sources
- Static qubit - electron spin
  - ✓ Long electron spin times  $T_1 \sim \text{ms}$ ,  $T_2 \sim \mu\text{s}$
  - ✓ Fast electron-spin cooling / manipulation
  - ✓ Long hole spin times  $T_1 \sim \text{ms}$ ,  $T_2 > 100\text{ns}$
  - ✓ Fast hole-spin cooling / manipulation
- Quantum gates
  - Photon-photon /spin-spin interactions**
    - Turchette et al, PRL 75, 4710(95)
    - Loss et al, PRA 57, 120 (98)
    - Imamoglu et al, PRL 83, 4204 (98)
    - Calarco et al, PRA 68, 012310(03)
  - Photon-spin interaction**
    - Duan and Kimble, PRL 92,127902(04)
    - Yao et al, PRL 95, 030504(05)
    - Bonato et al, PRL104, 160503(10)
    - Hu et al, PRB 78, 085307 (08); *ibid* 78, 125318 (08)
    - Hu et al, PRB 80, 205326(09)
    - Hu and Rarity, PRB 83, 115303(11)

Our work  $\Rightarrow$



## Outline

- Giant optical Faraday rotation / Giant circular birefringence
- Photon-spin entangling gates: universal, deterministic, fast (~tens ps)  
Conditional phase gate (type I)

$$\hat{U}(\pi/2) = e^{i\frac{\pi}{2}(|L\rangle\langle L|\otimes|\uparrow\rangle\langle\downarrow| + |R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow|)}$$

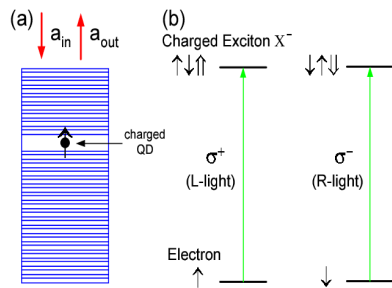
Entanglement beam splitter (type II)

$$\hat{i} = |R\rangle\langle R|\otimes|\uparrow\rangle\langle\uparrow| + |L\rangle\langle L|\otimes|\downarrow\rangle\langle\downarrow|$$

$$\hat{r} = |R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow| + |L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow|$$

- Single-photon based spin measurement, preparation, control
- Entanglement generation: photon-spin /spin-spin /photon-photon
- Photon-spin interfaces /spin memory
- Complete Bell-state analysers
- On-chip quantum repeaters
- Loophole-free Bell test
- Single-photon devices: switch, isolator, circulator, modulator, ...
- Conclusions

## Giant Faraday rotation & photon-spin entangling gate (type I)

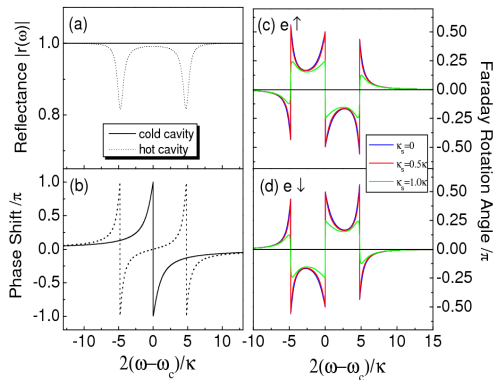


### Heisenberg equations of motion

$$\begin{cases} \frac{d\hat{a}}{dt} = -[i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}]\hat{a} - g\sigma_- - \sqrt{\kappa}\hat{a}_{in} + \hat{h} \\ \frac{d\sigma_-}{dt} = -[i(\omega_a - \omega) + \frac{\gamma}{2}]\sigma_- - g\sigma_z\hat{a} + \hat{f} \\ \hat{a}_{out} = \hat{a}_{in} + \sqrt{\kappa}\hat{a} \end{cases}$$

### Reflection coefficient (under weak-excitation limit)

$$r(\omega) = 1 - \frac{\kappa[i(\omega_{ex} - \omega) + \frac{\gamma}{2}]}{[i(\omega_{ex} - \omega) + \frac{\gamma}{2}][i(\omega_c - \omega) + \frac{\kappa}{2}] + g^2}$$



Hu et al, Phys. Rev. B 78, 085307 (08)

### Giant optical Faraday rotation — single-spin magneto-optical effect

- Spin  $\uparrow$ , L-light feels a hot cavity and R-light feels a cold cavity
- Spin  $\downarrow$ , R-light feels a hot cavity and L-light feels a cold cavity
- Large phase difference between cold and hot cavity

$$\theta_F^\uparrow = \frac{\varphi_0 - \varphi}{2} = -\theta_F^\downarrow$$

- Switch, isolator, circulator, router ...
- Quantum gates

## Photon-spin entangling gate (type I)

### Reflection operator

$$\hat{r}(\omega) = |r_0(\omega)|e^{i\varphi_0}(|R\rangle\langle R|\otimes|\uparrow\rangle\langle\uparrow| + |L\rangle\langle L|\otimes|\downarrow\rangle\langle\downarrow|) + |r_h(\omega)|e^{i\varphi_h}(|R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow| + |L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow|)$$

### Phase shift operator [when $r_0(\omega) = r_h(\omega)$ ]

$$\hat{U}(\Delta\varphi) = e^{i\Delta\varphi(|L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow| + |R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow|)} \Rightarrow \hat{U}(\pi/2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Delta\varphi$  is tunable between  $[-\pi, +\pi]$

$\Delta\varphi = \pi/2$  can be achieved in Purcell  
and strong coupling regime if  $\kappa_s/\kappa < 1.3$

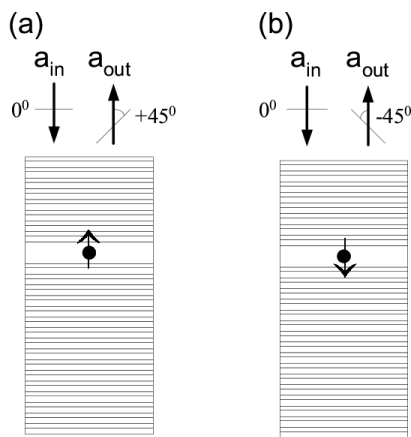
Ideal quantum measurement of spin

### Gate features

- ✓ Universal (conditional phase gate, or parity gate)
- ✓ Unity efficiency
- ✓ High fidelity
- ✓ Fast ( $\sim$  tens ps)  $\ll$  spin decoherence time ( $\sim \mu$ s)

Hu et al, Phys. Rev. B 78, 085307 (08)

## Quantum non-demolition measurement of spin (type I)



$$\text{Input photon } |H\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$$

Spin  $\uparrow$

$$|H\rangle \otimes |\uparrow\rangle \xrightarrow{\hat{U}(\pi/2)} \frac{1}{\sqrt{2}}|+45^\circ\rangle |\uparrow\rangle$$

Spin  $\downarrow$

$$|H\rangle \otimes |\downarrow\rangle \xrightarrow{\hat{U}(\pi/2)} \frac{1}{\sqrt{2}}|-45^\circ\rangle |\downarrow\rangle$$

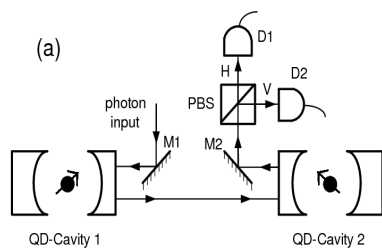
Spin superposition state  $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$$|H\rangle \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \xrightarrow{\hat{U}(\pi/2)} \frac{1}{\sqrt{2}}(\alpha|+45^\circ\rangle |\uparrow\rangle + \beta|-45^\circ\rangle |\downarrow\rangle)$$

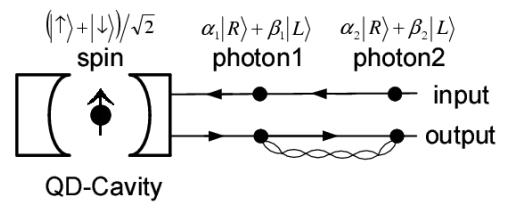
**Photon-spin entangler!**

- Unity efficiency and fast (~tens ps)
- Single-photon based
- Spin projective measurement
- Spin initialisation
- Quantum feedback control

## Spin entangler

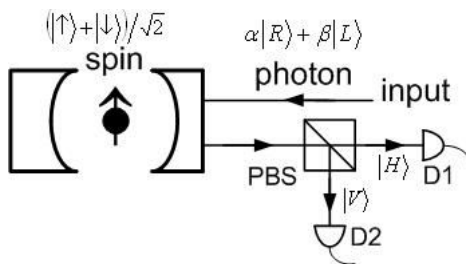


## Photon entangler

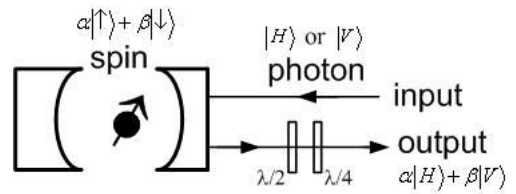


## Photon-spin quantum interface / spin memory

(a) Write in

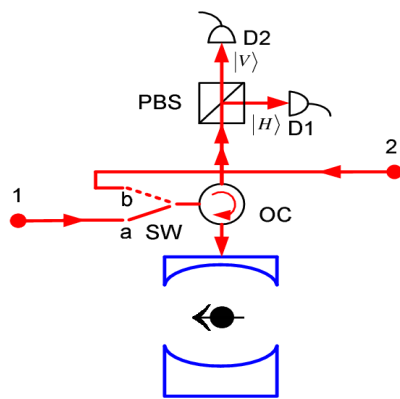


(b) Read out



Hu et al, Phys. Rev. B 78, 085307 (08); 78, 125318(08)

## Complete Bell-state analyzer (type I)



### Four Bell States

$$|\Psi^\pm\rangle = (|R\rangle_1|L\rangle_2 \pm |L\rangle_1|R\rangle_2)/\sqrt{2}$$

$$|\Phi^\pm\rangle = (|R\rangle_1|R\rangle_2 \pm |L\rangle_1|L\rangle_2)/\sqrt{2}$$

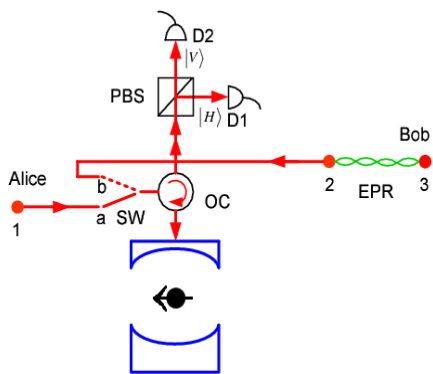
$$\hat{U}(\pi/2)|\Psi^\pm\rangle|+\rangle = i|\Psi^\pm\rangle|+\rangle$$

$$\hat{U}(\pi/2)|\Phi^\pm\rangle|+\rangle = |\Phi^\mp\rangle|-\rangle$$

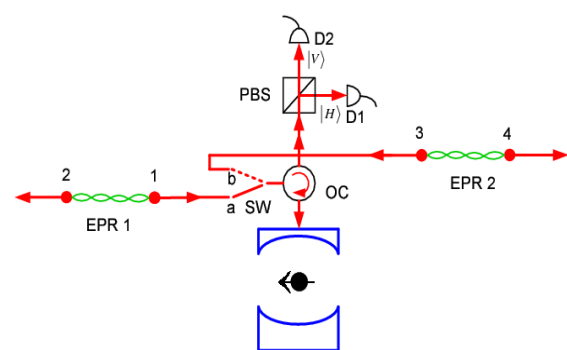
- ✓ Spin measurements check parity  $|\Psi^\pm\rangle$  and  $|\Phi^\pm\rangle$   
Polarization measurements check phase  $|\Psi^+\rangle$  and  $|\Psi^-\rangle$ ,  $|\Phi^+\rangle$  and  $|\Phi^-\rangle$
- ✓ Complete and loss-resistant due to built-in spin memory
- ✓ No photon synchronization, no indistinguishability
- ✓ Global quantum networks via satellites



## Loss-resistant teleportation and entanglement swapping



Photons 1, 2	Spin	Photon 3
$ H\rangle_1 H\rangle_2$ or $ V\rangle_1 V\rangle_2$	$ -\rangle$	$\alpha L\rangle_3 - \beta R\rangle_3$
$ H\rangle_1 V\rangle_2$ or $ V\rangle_1 H\rangle_2$	$ -\rangle$	$\alpha L\rangle_3 + \beta R\rangle_3$
$ H\rangle_1 H\rangle_2$ or $ V\rangle_1 V\rangle_2$	$ +\rangle$	$\alpha R\rangle_3 + \beta L\rangle_3$
$ H\rangle_1 V\rangle_2$ or $ V\rangle_1 H\rangle_2$	$ +\rangle$	$\alpha R\rangle_3 - \beta L\rangle_3$



Photons 1, 3	Spin	Photons 2, 4
$ H\rangle_1 H\rangle_3$ or $ V\rangle_1 V\rangle_3$	$ -\rangle$	$[ L\rangle_2 L\rangle_4 -  R\rangle_2 R\rangle_4]/\sqrt{2}$
$ H\rangle_1 V\rangle_3$ or $ V\rangle_1 H\rangle_3$	$ -\rangle$	$[ L\rangle_2 L\rangle_4 +  R\rangle_2 R\rangle_4]/\sqrt{2}$
$ H\rangle_1 H\rangle_3$ or $ V\rangle_1 V\rangle_3$	$ +\rangle$	$[ L\rangle_2 R\rangle_4 +  R\rangle_2 L\rangle_4]/\sqrt{2}$
$ H\rangle_1 V\rangle_3$ or $ V\rangle_1 H\rangle_3$	$ +\rangle$	$[ L\rangle_2 R\rangle_4 -  R\rangle_2 L\rangle_4]/\sqrt{2}$

### Three-step teleportation process

- (a) On detecting photon 1, photon 1 state is transferred to spin
- (b) On detecting photon 2, spin and photon 3 get entangled
- (c) On measuring spin, spin state is transferred to photon 3

Hu and Rarity, Phys. Rev. B 83, 115303(11)

## Single-photon based spin control (type I)

- Spin in  $\uparrow, \downarrow$  states  $\rightarrow$  giant optical Faraday rotation
- Photon in R, L states  $\rightarrow$  giant spin rotation

one photon pulse in  $|R\rangle$  or  $|L\rangle$  —  $\pi/2$  pulse

$$\begin{aligned}\hat{U}(\pi/2)|R\rangle(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) &= |R\rangle(\alpha|\uparrow\rangle + i\beta|\downarrow\rangle) \\ \hat{U}(\pi/2)|L\rangle(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) &= |L\rangle(i\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\end{aligned}$$

two photon pulses in  $|R\rangle$  or  $|L\rangle$  —  $\pi$  pulse

$$\begin{aligned}\hat{U}(\pi/2)|R\rangle_1|R\rangle_2(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) &= |R\rangle_1|R\rangle_2(\alpha|\uparrow\rangle - \beta|\downarrow\rangle) \\ \hat{U}(\pi/2)|L\rangle_1|L\rangle_2(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) &= |L\rangle_1|L\rangle_2(-\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\end{aligned}$$

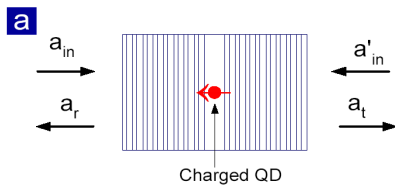
- Spin echo/dynamic decoupling to preserve the spin coherence

$$\left(\frac{\pi}{2}\right)_x - (\pi)_z - \left(\frac{\pi}{2}\right)_x \quad \text{compatible with QIP protocols}$$

Spin rotation around z with single photons

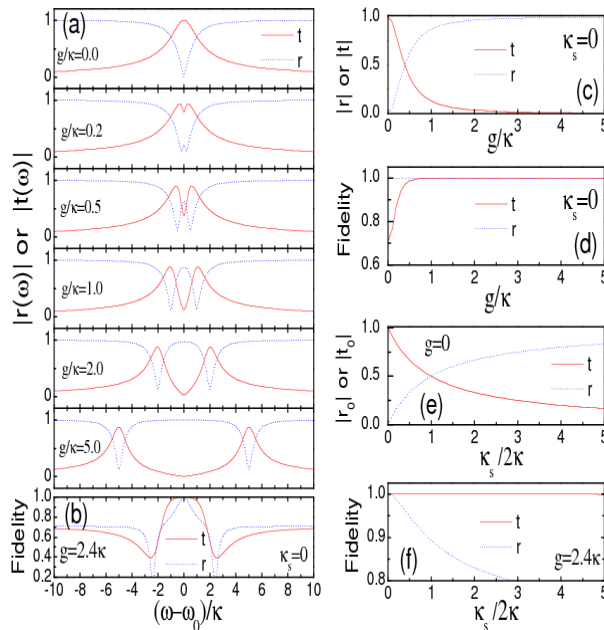
Spin rotation around x with laser pulse (optical Stark effect) or B in Voigt

## Giant circular birefringence & photon-spin entangling gate II



universal, deterministic, fast (~tens ps)

In Purcell or strong coupling regime



If ↑, R-photon transmitted  
L-photon reflected

If ↓, R-photon reflected  
L-photon transmitted

Transmission operator

$$\hat{t}(\omega) \approx t_0(\omega) (|R\rangle\langle R| \otimes |\uparrow\rangle\langle\uparrow| + |L\rangle\langle L| \otimes |\downarrow\rangle\langle\downarrow|)$$

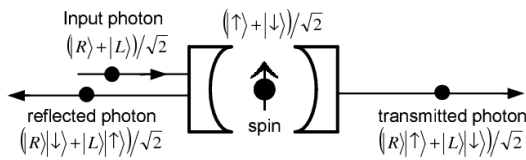
Reflection operator

$$\hat{r}(\omega) \approx r(\omega) (|R\rangle\langle R| \otimes |\downarrow\rangle\langle\downarrow| + |L\rangle\langle L| \otimes |\uparrow\rangle\langle\uparrow|)$$

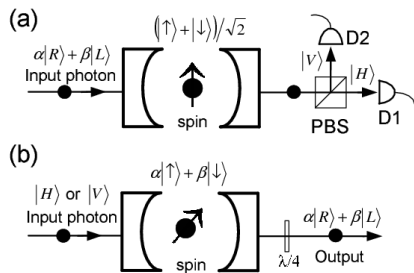
$$t_0(\omega_0) \approx -1, r(\omega_0) \approx 1 \quad \text{if } \kappa_s \ll \kappa$$



### Photon-spin entangler – Entanglement beam splitter

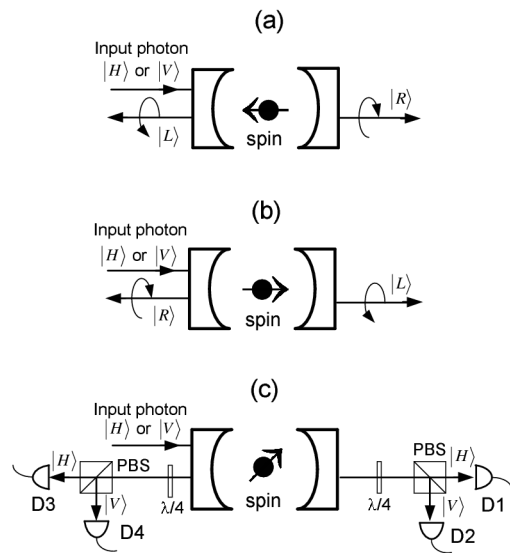


### Photon-spin interface



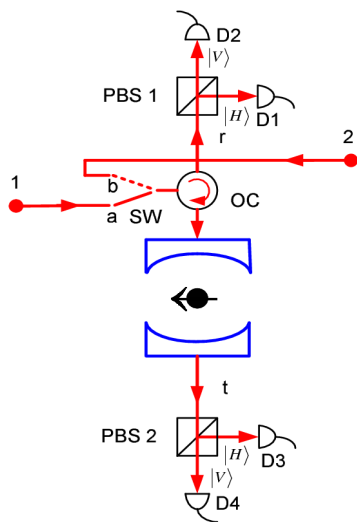
Hu et al , PRB 80, 205326(09)

### QND measurement of spin



Spin entangler  
Photon entangler

## Complete Bell-state Analyzer (type II)



- ✓ Path measurements check parity  $|\Psi^\pm\rangle$  and  $|\Phi^\pm\rangle$   
Polarization measurements check phase  $|\Psi^+\rangle$  and  $|\Psi^-\rangle$ ,  $|\Phi^+\rangle$  and  $|\Phi^-\rangle$
- ✓ Complete and loss-resistant due to built-in spin memory
- ✓ No photon synchronization, no indistinguishability
- ✓ Global quantum networks via satellites

$$|\Psi^\pm\rangle|+\rangle \xrightarrow{\vec{r}, \vec{t}} -\left[|R\rangle_1^t |L\rangle_2^r \pm |L\rangle_1^r |L\rangle_2^t\right]|\uparrow\rangle - \left[|R\rangle_1^r |L\rangle_2^t \pm |L\rangle_1^t |L\rangle_2^r\right]|\downarrow\rangle$$

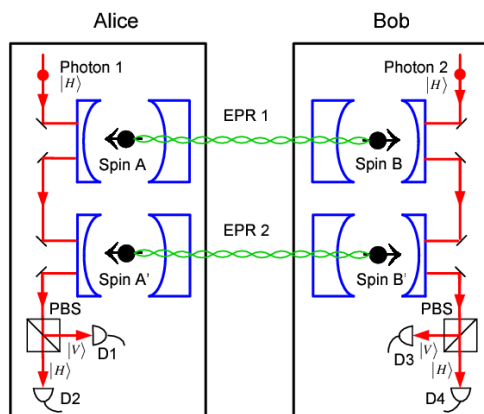
$$|\Phi^\pm\rangle|+\rangle \xrightarrow{\vec{r}, \vec{t}} \left[|R\rangle_1^t |R\rangle_2^t \pm |L\rangle_1^r |L\rangle_2^r\right]|\uparrow\rangle + \left[|R\rangle_1^r |R\rangle_2^r \pm |L\rangle_1^t |L\rangle_2^t\right]|\downarrow\rangle$$

Hu and Rarity, Phys. Rev. B 83, 115303(11)

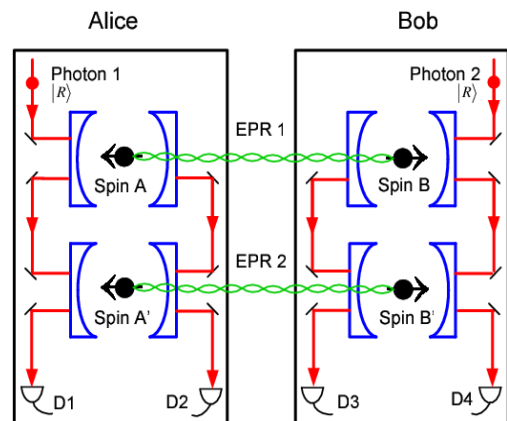
## On-chip quantum repeaters (two types)

- Spin-cavity units as quantum repeaters  
Entanglement generation, swapping, purification and storage can all be performed with the spin-cavity units
- Global quantum networks via quantum repeaters

### Purification (type-I)



### Purification (type-II)

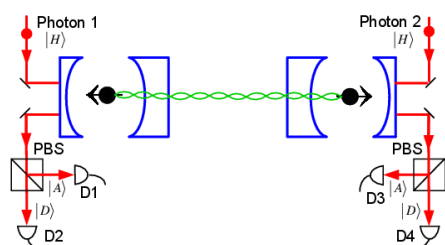


## Challenges for entanglement purification

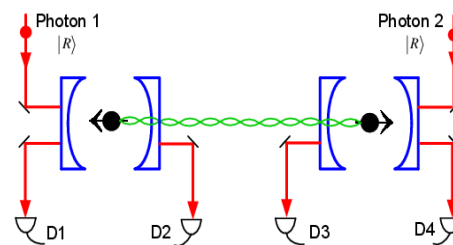
- Quantum repeaters need gate errors on the percent level
- Gate imperfections
  - Imperfect spin selection rules (error <10%)
  - Side-leakage from cavity
  - Short spin dephasing time ( $\sim$ ns)
  - Solutions: spin echo/dynamic decoupling
  - electron spin  $\rightarrow$  hole spin

## Loophole-free Bell test (two types)

type-I



type-II



- **Generate spin-spin entanglement**  
via a single photon or  
photon-photon entanglement → spin-spin entanglement
- **Close detection loophole**  
Single-photon based spin measurement with unity efficiency
- **Close locality loophole**  
Spin measurement time  $\Delta t <$  signal transfer time  $d/c$   
 $\Delta t \sim$  tens ps,  $d > 3\text{mm}$
- **Device independent protocols, such as DIQKD, certified random number generation**



## Comparison of different cavity-QED systems with type-I gate in non-ideal case $U(\Delta\phi)$

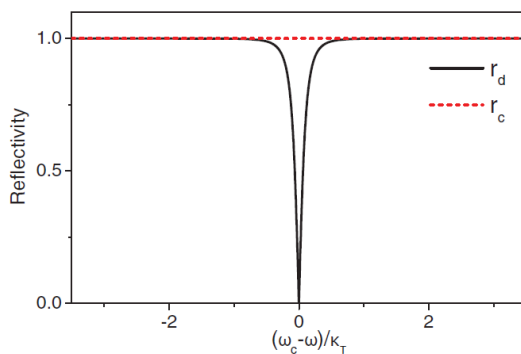
System	Atoms	N-V centres	Dots	Low-Q
$g/(2\pi)$ (MHz)	5	100	5000	3300
$\kappa/(2\pi)$ (MHz)	3	13	3000	440000
$\kappa_s/(2\pi)$ (MHz)	0.5	39	7000	220000
$\gamma/(2\pi)$ (MHz)	3	0.6	1000	6
$\bar{\alpha}$ (Rad)	$\sim 0.4\pi$	$\sim 0.1\pi$	$\sim 0.1\pi$	$\sim 0.4\pi$
$\tau$ ( $\mu s$ )	10000	1000	1	1000
$\Delta t$ (ns)	500	300	1.5	1000
Min $D$ (m)	150	100	$< 1$	300
Min $t$ ( $\mu s$ )	$\sim 1$	$\sim 0.3$	$\sim 0.1$	$\sim 2$

QD spin-cavity system is better than atom or NV for loophole-free Bell test

N. Brunner et al, arXiv: 1303.6522(quant-ph)

## Entanglement generation with low-Q microcavities (single-sided)

## Non-deterministic scheme



Resonant scattering regime

$$g^2 = \kappa\gamma/4 \text{ at } \omega = \omega_0$$

$$r_d = 0$$

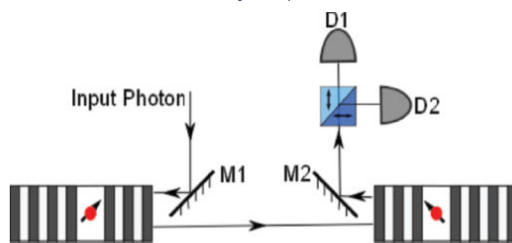
$$r_c = 1$$

$$|R\rangle \otimes |\uparrow\rangle \rightarrow r_d |R\rangle |\uparrow\rangle,$$

$$|R\rangle \otimes |\downarrow\rangle \rightarrow r_c |R\rangle |\downarrow\rangle,$$

$$|L\rangle \otimes |\uparrow\rangle \rightarrow r_c |L\rangle |\uparrow\rangle,$$

$$|L\rangle \otimes |\downarrow\rangle \rightarrow r_d |L\rangle |\downarrow\rangle.$$



Suitable for QD-spin, NV

Advantage: low-Q, easy to implement

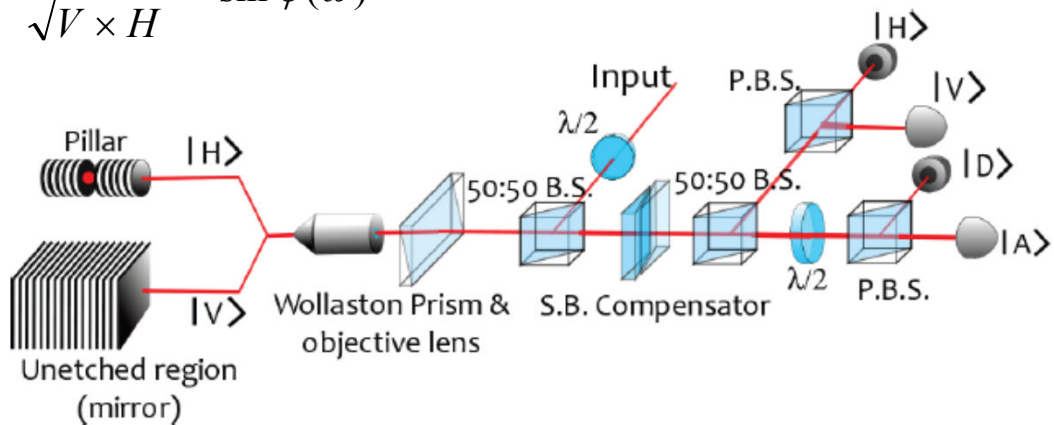
Disadvantage: non-deterministic

Young et al, Phys. Rev. A 87, 012332(11)

## Recent experimental progress with uncharged dots towards type-I gate

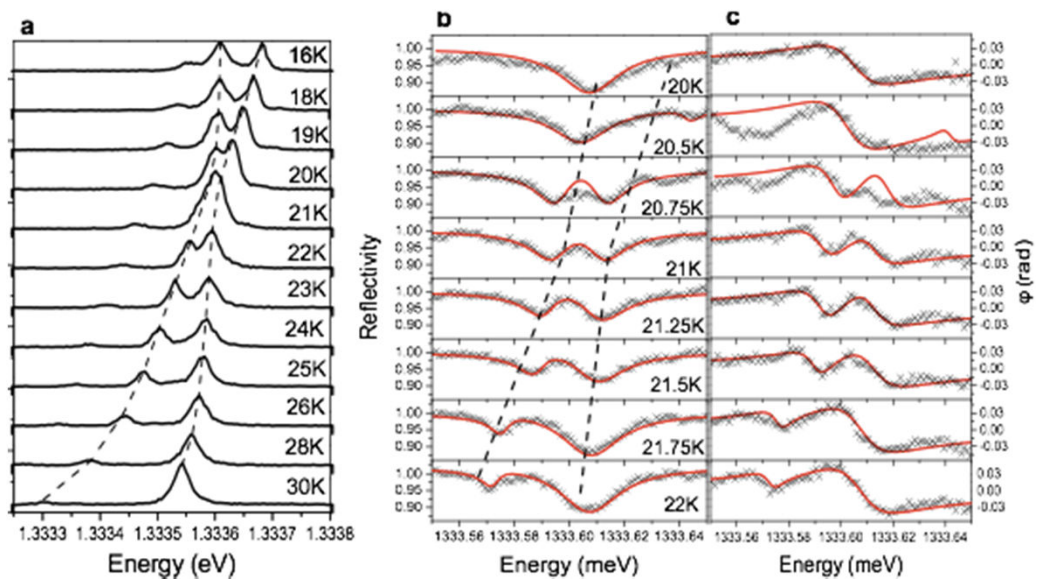
Reflection spectroscopy  
— Conditional phase shift interferometer

$$\frac{D - A}{\sqrt{V \times H}} = \sin \phi(\omega)$$



Young et al, Phys. Rev. A 84, 011803(R) (2011)

## Comparing PL and resonant spectroscopy

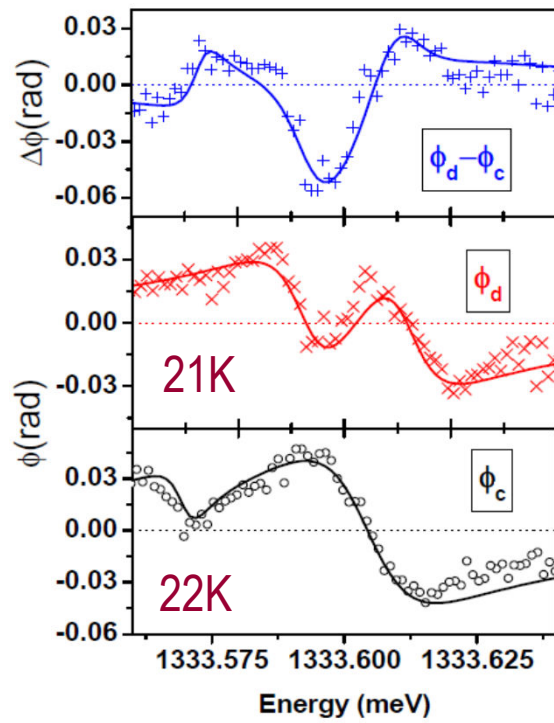
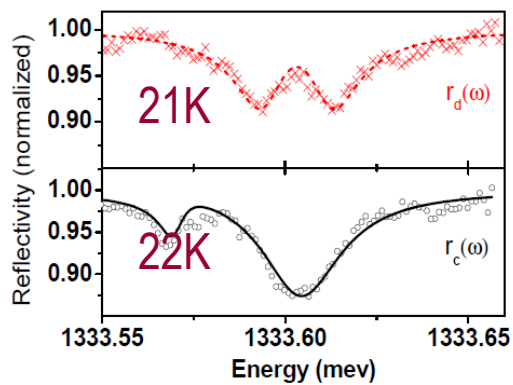


Young et al, Phys. Rev. A 84, 011803(R) (2011)

## Conditional phase of $3^\circ$ observed between a dot on and off-resonance with cavity

$$g \sim 9.4 \mu\text{eV} \quad \kappa + \kappa_S \sim 26 \mu\text{eV} \quad \gamma \sim 5 \mu\text{eV}$$

$$g > (\kappa + \kappa_S + \gamma)/4 \quad \Delta\phi \sim 3^\circ$$

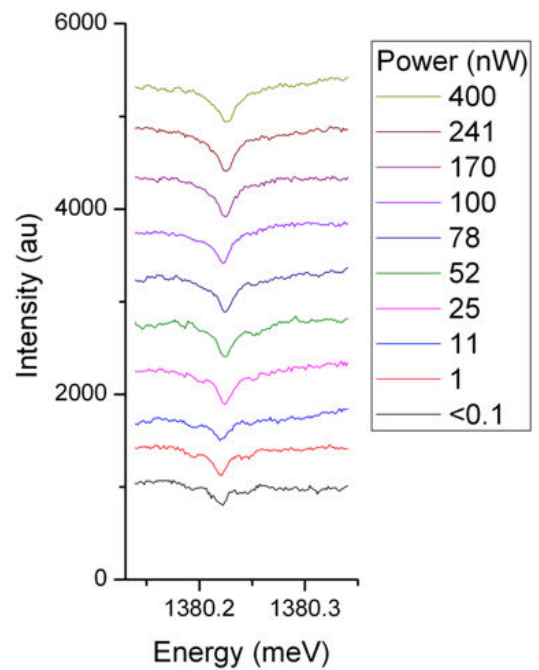


Young et al, Phys. Rev. A 84, 011803(R) (2011)

## Latest results from the laboratory

### Optical nonlinearity at $\sim 10$ photons per cavity lifetime

- Dot on resonance with a cavity studied in reflection
- We see a dot-cavity feature which saturates at 100nW
- If this is strongly coupled the timescale is that of the cavity decay, ie  $\sim 30$ ps
- Thus 100nW in 30ps gives optical nonlinearity at  $\sim 10$  photons per cavity lifetime



## Requirements to realize the photon-spin gates

- ✓ Charged QDs
  - Modulation-doping
  - Distinguish between charged and neutral excitons is non-trivial.
- ✓ High-quality microcavity
  - Weak and strong coupling
  - Low side leakage
- ✓ Spin initialization via spin measurement with single photons
- ✓ Spin control
  - Rotation around z with single photons
  - Rotation around x with laser pulse (optical Stark effect) or B in Voigt
- ✓ Spin echo or dynamical decoupling to preserve spin coherence
  - Compatible with QIP protocols

## Summary and Outlook

- **Two spin-cavity units — photon-spin entangling gates**
  - ✓ Conditional photon-spin interaction
  - ✓ GFR and GCB are **optical linear effects**
  - ✓ Universal
  - ✓ Deterministic (if optimized)
  - ✓ Fast (~tens ps)
    - Spin coherence time ( $\mu\text{s}$ ) > 100,000 gate time
  - ✓ Single-photon based spin initialisation, measurement, and control
  - ✓ BSM, repeaters, photon-spin interface/spin memory
  - ✓ Realizable with current semiconductor technology
    - Q=40,000 for  $d=1.5 \mu\text{m}$  micropillars, Reitzenstein et al., APL 90, 251109 (07)
    - Purcell enhancement and strong coupling are achieved
- **Global and secure quantum networks**
  - ✓ Via satellites or quantum repeaters
  - ✓ Detection and locality loopholes can be closed simultaneously
- **Practical solid-state quantum computers are possible**
  - DiVincenzo's criteria are all met!
- **Single-photon devices: switch, isolator, circulator, router, ...**
- **Spin-cavity units can be applied in all aspects of QIP**



## Acknowledgements

In collaborations with

William J. Munro (NTT, Japan)

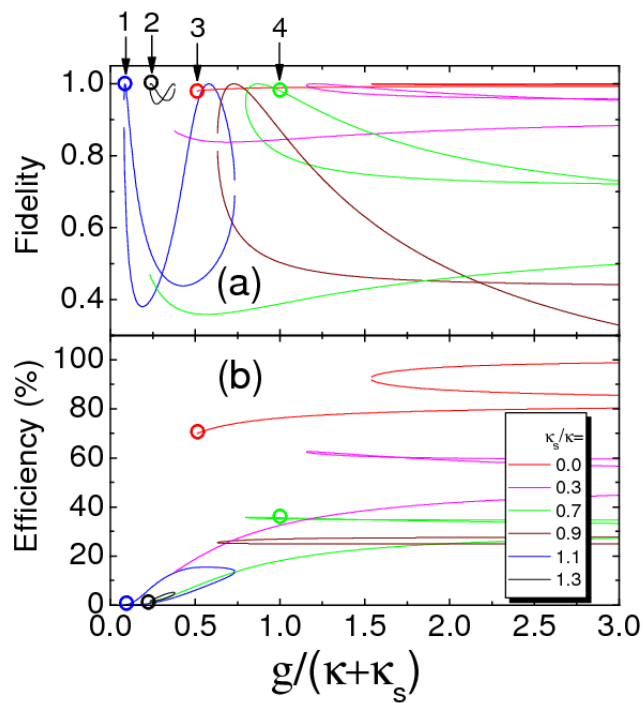
Jeremy L. O'Brien (CQP, Bristol)

Nicolas Brunner (Bristol / Geneva)

Thanks for your attention!



## Fidelity and Efficiency of BSA (Type I)



Hu and Rarity, Phys. Rev. B 83, 115303(11)



$$F^{(\Psi^\pm)} = 1$$

$$F^{(\Phi^\pm)} = \frac{1}{\sqrt{1 + \frac{1}{4} \left( \left| \frac{r_o(\omega)}{r_h(\omega)} \right| + \left| \frac{r_h(\omega)}{r_o(\omega)} \right| \right)^2}}$$

$$\eta = \frac{1}{4} \left( |r_o(\omega')|^2 + |r_h(\omega')|^2 \right)^2$$