Invited Talks

Title: Graphs with three eigenvalues and finite geometries called *GQ*
Speaker: Aleksandar Jurišic *(Ljubljana)*
Abstract: A graph with three eigenvalues is a union of complete graphs of the same size or its complement or a graph of diameter 2, with the number of common neighbours of a pair of vertices depending only on their distance (0, 1 or 2). We study such graphs (known under the name strongly regular graphs) and their generalizations (distance-regular graphs) in connection to some special finite geometries known as generalized quadrangles. They can be used to construct incredible objects (from small to large - even with more than quarter of million vertices) and then even characterized by a very small number of regularity properties.

Title: Promotion for staircase tableaux
Speaker: Kevin Purbhoo *(Waterloo)*
Abstract: Promotion is an invertible operation on standard young tableaux. It has received a fair bit of attention recently because of its relevance in representation theory, and its intriguing combinatorial structure. In 2010, Rhoades proved a theorem that the describes the orbit structure of promotion on tableaux of rectangular shape, and since then there have been several other proofs of this result. In this talk, I will discuss some new results about promotion on staircase shapes. These are proved using geometric ideas, and are essentially attributable to the existence of some very special subvarieties of a Grassmannian.

Title: Trivial words in groups - the importance of counting nothing
Speaker: Andrew Rechnitzer
Abstract: Random walks appear at the heart of many problems in mathematics. Perhaps one of the most famous questions is "What is the probability that a random walk returns to its starting point?"

For a random walk on the line or the square-grid, this question can be answered quite directly by recasting the problem as one of counting loops. However on more complicated graphs the problem is far from trivial. In the setting of geometric group theory, this question is intimately tied to the problem of "amenability" and the number of trivial words. While amenability (and so the probability that a random walker returns) can be decided for many groups, it remains "very open" for Thompson's group F.

In this work, we apply numerical and enumerative methods from statistical mechanics and combinatorics to the study of random walks on groups and so examine the amenability of Thompson's group.

This is work together with Murray Elder, Buks van Rensburg and Thomas Wong.

Contributed Talks

Title: Game-Theoretic Network Formation
Speaker: Omid Atabati *(Brock)*
Abstract: We study the dynamics of a game-theoretic network formation model that yields large-scale small-world networks. So far, mostly stochastic frameworks have been utilized to explain the emergence of these networks. In particular, Kleinberg in [2] proposed a stochastic grid-based model that is a generalization of small-world model ofWatts and Strogatz in [3] and was first to generate a family of random networks that possesses paramount properties of social networks. On the other hand, it is natural to seek for game-theoretic network formation models in which links are formed due to strategic behaviors of individuals, rather than based on probabilities. This direction is in its early attempts. Even-dar and Kearns in [1] constructed a game theoretic model following by an earlier grid-based stochastic model but an economic contrast to it. In this linking game, a player i can purchase a link to another player j at grid distance d at the "fixed" cost of d^{α}. Also players seek to minimize their distances to other players. This model demonstrates a sharp threshold for the diameter of the network: for any $\alpha < 2$, the network has a constant diameter while for any $\alpha > 2$, the diameter grows as a root of the network size. Their model assumes unilateral formation and severance for links and also relaxes the notion of Nash equilibrium to link stability. Inspired by this model, we consider a more realistic framework. We define dynamic link-prices and maintenance costs for link establishments, and allow players to put transfer payments on links. We demonstrate a small constant diameter in all equilibrium networks in our model for any $\alpha > 0$. Unlike the earlier model, we show that not only the existence of equilibrium networks is guaranteed in our model, but also, these networks coincide with the outcomes of Nash equilibrium in network formation. We also analyze the impact of positing players in a topic-based setting by constructing a strategic model in hierarchical network within a complete b-ary tree as the seed network.

References

[1] E Even-Dar, M Kearns, 2007, "A small world threshold for economic network formation," Advances in Neural Information Processing Systems 19, Pg 385-392.

[2] J. Kleinberg, 2000, "The small-world phenomenon: An algorithmic perspective", Symposium on the Theory of Computing 32, Pages 163-170.

[3] D.J. Watts and S.H. Strogatz, 1998, "Collective dynamics of small-world networks", Nature 393, Pages 440-442.

Title: The Total Acquisition Number of Random Graphs
Speaker: Deepak Bal *(Ryerson)*
Abstract: Let G be a graph in which each vertex initially has weight 1. In each step, the weight from a vertex u can be moved to a neighbouring vertex v, provided that the weight on v is at least as large as the weight on u. The total acquisition number of G, denoted by $a_t(G)$, is the minimum possible size of the set of vertices with positive weight at the end of the process. In this talk, we discuss results on the value of $a_t(G)$ for random graphs.

Title: A Schur-Like Basis of NSym Defined by a Pieri Rule
Speaker: John Campbell *(York)*
Abstract: Recent research on the algebra of non-commutative symmetric functions and the dual algebra of quasi-symmetric functions has explored some natural analogues of the Schur basis of the algebra of symmetric functions. We introduce a new basis of the algebra of non-commutative symmetric functions using a right Pieri rule. The commutative image of an element of this basis indexed by a partition equals the element of the Schur basis indexed by the same partition and the commutative image is 0 otherwise. We establish a rule for right-multiplying an arbitrary element of this basis by an arbitrary element of the ribbon basis, and a Murnaghan-Nakayama-like rule for this new basis. Elements of this new basis indexed by compositions of the form $(1^n, m, 1^r)$ are evaluated in terms of the complete homogeneous basis and the elementary basis.

Title: Recent Developments in Quantum State Transfer
Speaker: Gabriel Coutinho *(Waterloo)*
Abstract: I will introduce the topic of continuous-time quantum walks and explain how it motivates very interesting questions in algebraic graph theory. I will review some recent developments and also present some original results.

Title: Algorithms for Community Detection within Social Networks
Speaker: Christopher Hogan *(Wilfrid Laurier)*
Abstract: Community detection is an interdisciplinary field of research that focuses on uncovering groups, or clusters, of highly connected nodes within a complex network. Identification of these clusters can reveal which nodes are most influential, and which nodes are interdependent. In the context of Social Networks individuals are represented by nodes in a graph, and their relationships with one another are represented by edges, and in this way graph theory techniques and algorithms can be brought to bear on the problem. This method of analysis has been used to discover relationships between users on Facebook, Twitter, and many other popular social networks.

My research analyzes the Plasticity social network, a creation of The Smile Epidemic, a start-up company in Waterloo region. This network connects employees within a company and focuses on promoting and spreading happiness,

which is measured according to accepted standards in psychology. This creates a rich pool of additional data about the nodes and edges in the social network. I will present a brief overview of current and popular algorithms for clustering social network data, as well as a discussion of the unique aspects of community structure related to the Plasticity social network, and the distinctive opportunities this provides for edge weighting. This is joint work with Angèle Hamel.

Title: Bipartite Covers of Linear Cayley Graphs with only Five Eigenvalues
Speaker: Junbo Huang *(Waterloo)*
Abstract: Bipartite graphs with an abelian group of automorphisms acting regularly on each colour class can be used to construct interesting sets of complex unit vectors, due to works by Godsil and Roy. When such a graph has exactly five distinct eigenvalues, the resulting set of vectors satisfies the property that the inner product between any two distinct vectors has absolute value 0 or α ($\alpha \neq 0$). Such a set is a generalization of equiangular sets and mutually unbiased bases, which received much attention due to their applications in quantum information theory.

Bipartite covers of linear Cayley graphs are natural examples that possess abelian groups of automorphisms acting regularly on the colour classes. In this talk, I will present a geometric condition for a bipartite cover of a linear Cayley graph to have only five distinct eigenvalues.

Title: Bounds for Graph-Intersecting Packings
Speaker: Elizabeth Maltais *(Ottawa)*
Abstract: We give bounds for collections of partitions, or more generally for collections of packings, whose classes intersect according to the edges of a graph. Bollobas (1965) gave a bound on two families of subsets having certain intersection properties; his inequality can be viewed as a special case of our bounds for graph-intersecting collections of packings, using the complete graph K_2. We also apply our method to obtain new upper bounds on the number of columns in a covering array with a fixed number of rows and alphabet with $v \geq 3$ symbols. This is joint work with Lucia Moura and Mike Newman.

Title: Spread Codes and Their Role in Communication
Speaker: Felice Manganiello *(Clemson)*
Abstract: Codes are mathematical structures used for reliable communication. Good codes are explicit constructions satisfying some optimality conditions. In this talk we learn how codes based on subspaces can be used to achieve optimal communication over some networks. We further speak about spread codes, an explicit optimal algebraic construction of subspace codes.

Title: Stress Matrices and Realisations of Graphs on Surfaces
Speaker: Tony Nixon *(York)*
Abstract: A framework is a geometric realisation of a graph in d-dimensional space. A stress is an assignment of weights to the edges of a framework such that an equilibrium condition holds at each vertex. That is, the sum of the weights, scaled by the vector difference of the end-vertices, is 0. Associated to a stress is a natural symmetric matrix with rows summing to 0. Connelly showed that, if the framework is generic, this stress matrix having maximal rank is sufficient to guarantee the framework is globally rigid (unique up to translations and rotations). In this talk I will describe an analogue of Connelly's theorem for frameworks in 3-dimensions where the points are constrained to lie on 2-dimensional varieties.

This is joint work with Bill Jackson.

Title: Random 312-Avoiding Permutations
Speaker: Lerna Pehlivan *(Washington)*
Abstract: A permutation of $\{1, 2, ..., N\}$ is said to avoid 312 pattern if there is no subsequence of three elements of this permutation that appears at the same relative order as 312. Monte Carlo experiments reveal some features of random 312 avoiding permutations. In light of these experiments we determine some probabilities explicitly and we obtain asymptotic approximations to these probabilities for large N.

This is a joint work with Neal Madras.

Title: Percolation with Small Clusters in Random Graphs
Speaker: Mustazee Rahman *(Toronto)*
Abstract: We consider the problem of finding an induced subgraph in a random d-regular graph such that its components have bounded size as the size of the graph gets arbitrarily large. We show that for any threshold T, the largest size density of such an induced subgraph with component sizes bounded by T is at most 2(log d)/d for asymptotically large d. A matching lower bound is known for independent sets. An analogous result holds for sparse Erdos-Renyi graphs.

Title: Computational Complexity of Maxclique for Cayely Graphs
Speaker: Brendan Rooney *(Waterloo)*
Abstract: Maxclique is the problem of finding the largest complete subgraph of an input graph. It is well-known to be an NP-Hard problem for general graphs. In 1998, Codenotti et al. showed that Maxclique is NP-Hard for the class of circulant graphs. We show that Maxclique is NP-Hard for the class of Cayley graphs on the direct products of any fixed finite group G.

Title: Monochromatic Path Decompositions
Speaker: Daniel T. Soukup *(Toronto)*
Abstract: The aim of this talk is to discuss monochromatic path decompositions of edge coloured infinite complete graphs. This line of research was initiated by a paper of R. Rado [1] where he proved that for every infinite edge colouring of the complete graph on $\mathbb{N}$, one can partition the vertex set into monochromatic paths of different colours.

We review previously known results, both on finite and infinite graphs, and present new generalizations. In particular, for every $r, k \in \mathbb{N}$ and every r-edge colouring of the complete graph on $\mathbb{N}$, one can partition the vertex set into finitely many monochromatic kth powers of paths.

Furthermore, we look at the question whether Rado's result extends to complete graphs of *arbitrary* infinite size. This is a joint work with M. Elekes, L. Soukup and Z. Szentmiklóssy.

References

[1] Rado, Richard; *Monochromatic paths in graphs.* Advances in graph theory (Cambridge Combinatorial Conf., Trinity College, Cambridge, 1977). Ann. Discrete Math. 3 (1978), 191–194.

Title: Finding a Second Hamilton Cycle: the Missing Link
Speaker: Andrew Wagner *(Ottawa)*
Abstract: A uniquely hamiltonian graph is one that admits exactly one Hamilton cycle. In 1946, Smith proved that there are no 3-regular uniquely hamiltonian graphs, which result was extended by Thomason in 1978 to all regular graphs of odd degree. A conjecture made by Sheehan in 1975 suggests that there are no 4-regular uniquely hamiltonian graphs. If the conjecture is true, it can be extended to all regular graphs of even degree, which will mean that there are no non-trivial uniquely hamiltonian regular graphs.

In this talk, we will explore the techniques used to achieve the best known results. We also give a sufficient condition for finding a second Hamilton cycle in 4-regular graphs with a large automorphism group.

This is a joint work with Mateja Šajna.

Title: A Surprisingly Simple de Bruijn Sequence Construction
Speaker: Dennis Wong *(Guelph)*
Abstract: A *de Bruijn sequence* is a cyclic sequence of length 2^n where each substring of length n corresponds to a unique string in the set of length n binary strings. The number of unique de Bruijn sequences for a given n is a whopping $2^{2^{n-1}}/2^n$; however, only a very small number are known to be constructed efficiently and practically (using a polynomial amount of space). This small number of known efficient de Bruijn sequence constructions is perhaps best captured by a quote from Fredericksen from 1982:

> When a mathematician on the street is presented with the problem of generating a full cycle (de Bruijn sequence), one of the three things happens: he gives up, or produces a sequence based on a primitive polynomial, or produces the prefer-one sequence. Only rarely is a new algorithm proposed.

In this talk, we introduce a novel shift-based construction to produce a *new* de Bruijn sequence.

Title: Uniform Mixing in Quantum Walks
Speaker: Harmony Zhan *(Waterloo)*
Abstract: A quantum walk on a graph X is determined by the transition matrix $U(t) = exp(itA)$. We say X admits uniform mixing at time t if all entries of $U(t)$ have the same absolute value. In this talk, we discuss the uniform mixing properties, and summarize some known results on graphs with uniform mixing and graphs resistant to uniform mixing.