Energy Storage: A problem at the intersection of Finance and Optimization

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WPI Financial Engineering for Energy and Commodity Risk Management, Sept 17 2012
Vienna, Austria.

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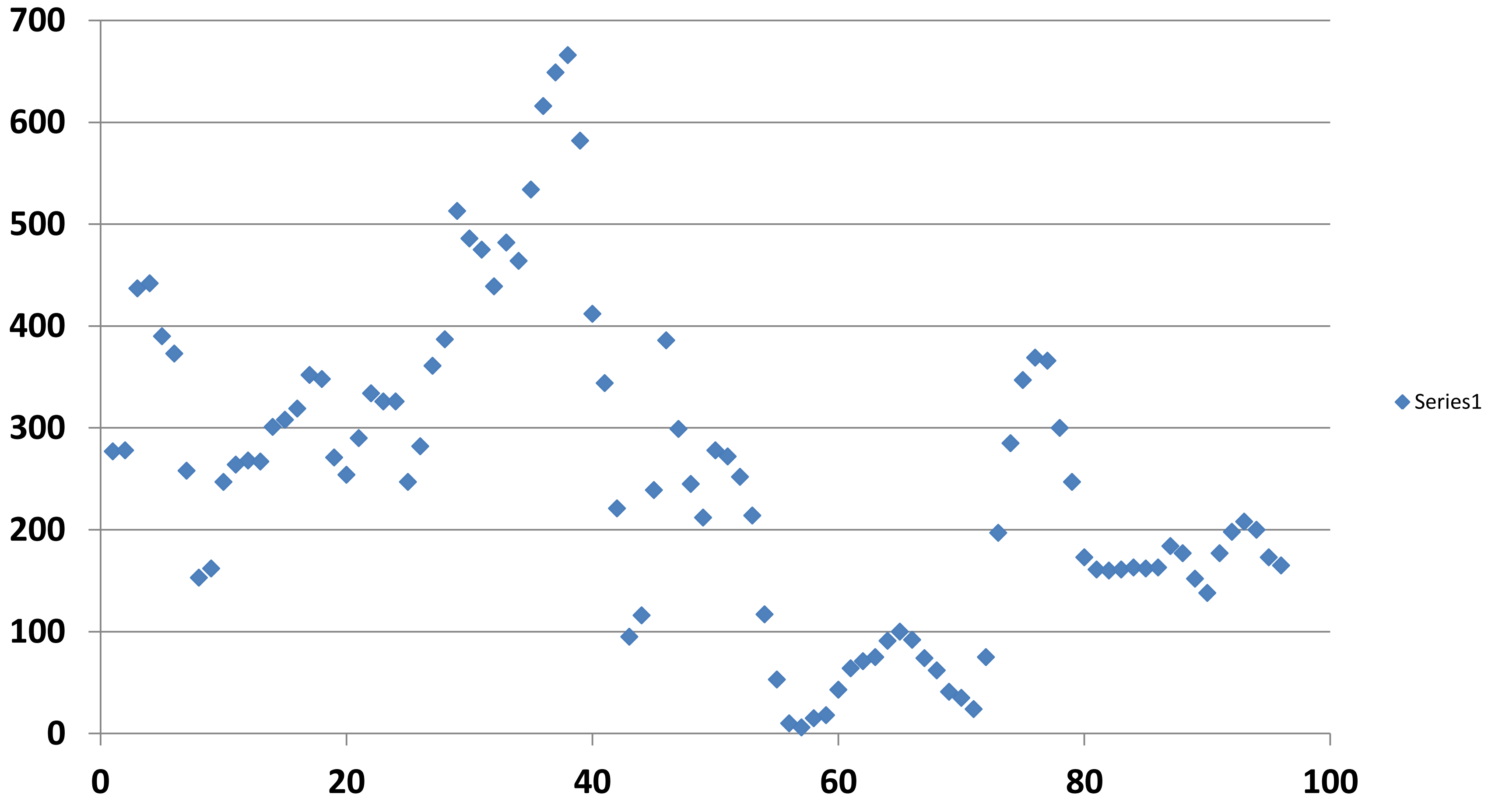
The problem

- Renewables (Wind, Solar, small Hydro) are the cornerstone of green power initiatives both in Ontario and worldwide.
- Wind “penetration” has increased dramatically in recent years
- But wind and other renewables require expensive subsidies and do not result in dispatchable power.

Hourly Ontario wind production (MW):

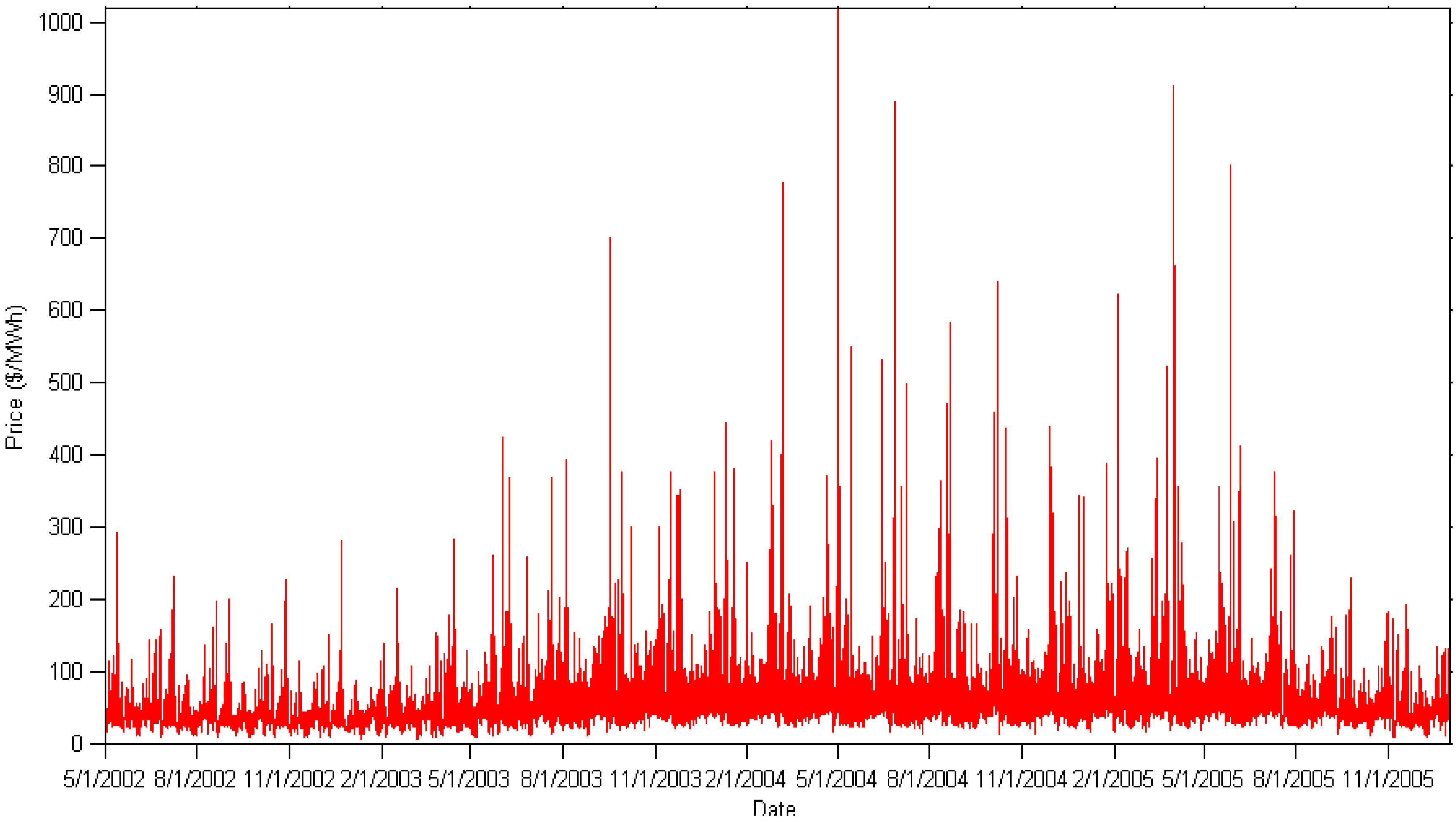
May 5 – May 8 2011. Source: IESO, Melissa Mielkie



Some see impacts of this on markets

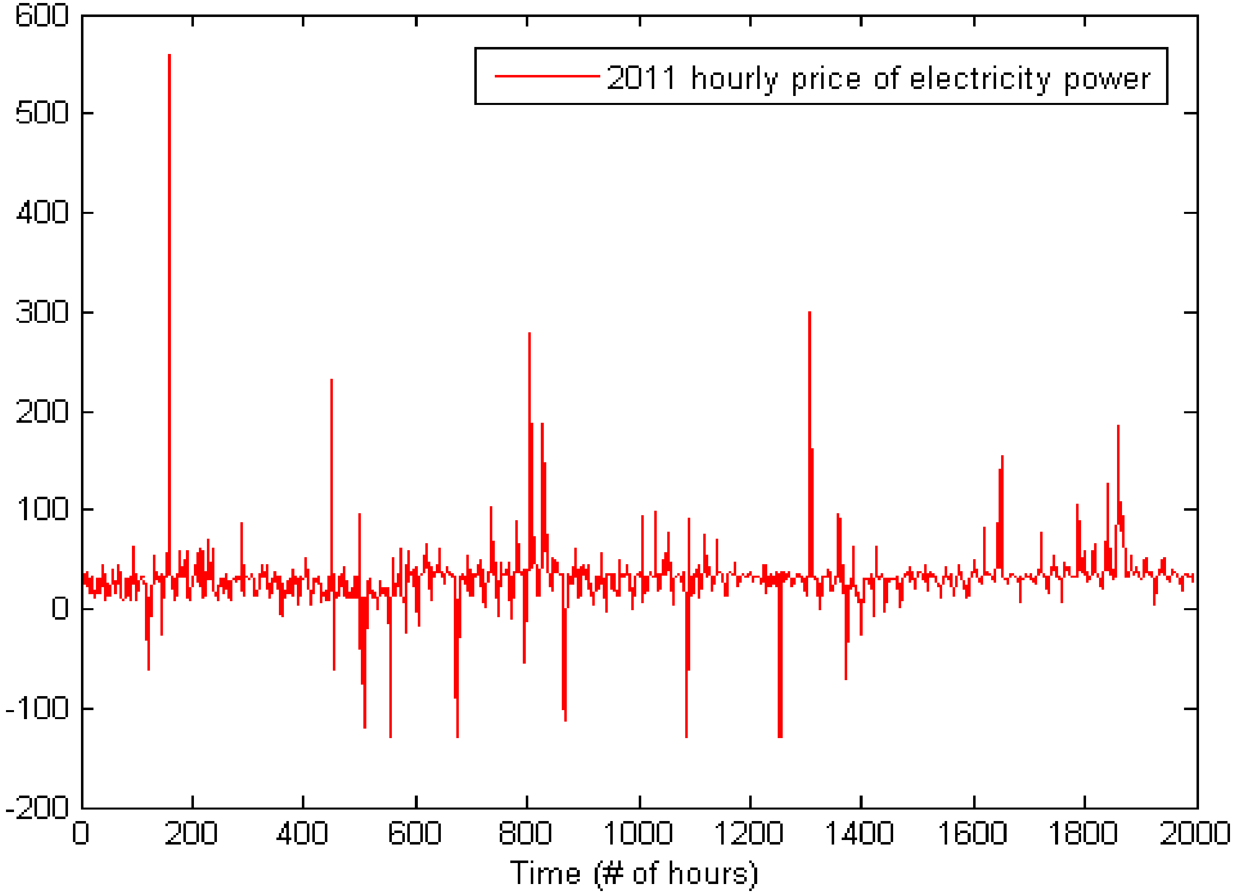
- Greater price instability
- Frequent negative prices

Ontario Open Market Price: Old days



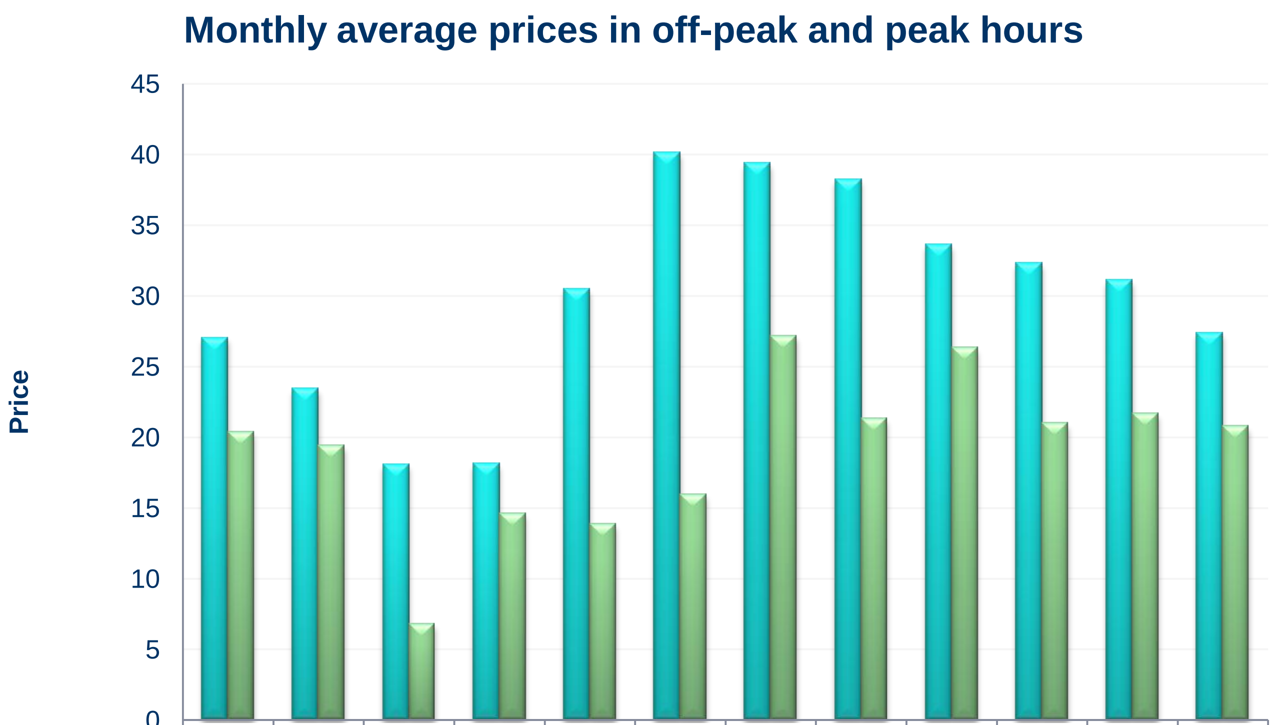
Ontario Electricity Price, $/MWh

Source IESO,



Price summary

- Prices show calendar year seasonality



	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
8AM-10PM	27.051	23.429	18.087	18.107	30.481	40.151	39.352	38.249	33.612	32.308	31.107	27.384
11PM-7AM	20.383	19.396	6.8257	14.605	13.873	15.975	27.168	21.355	26.322	20.979	21.69	20.764

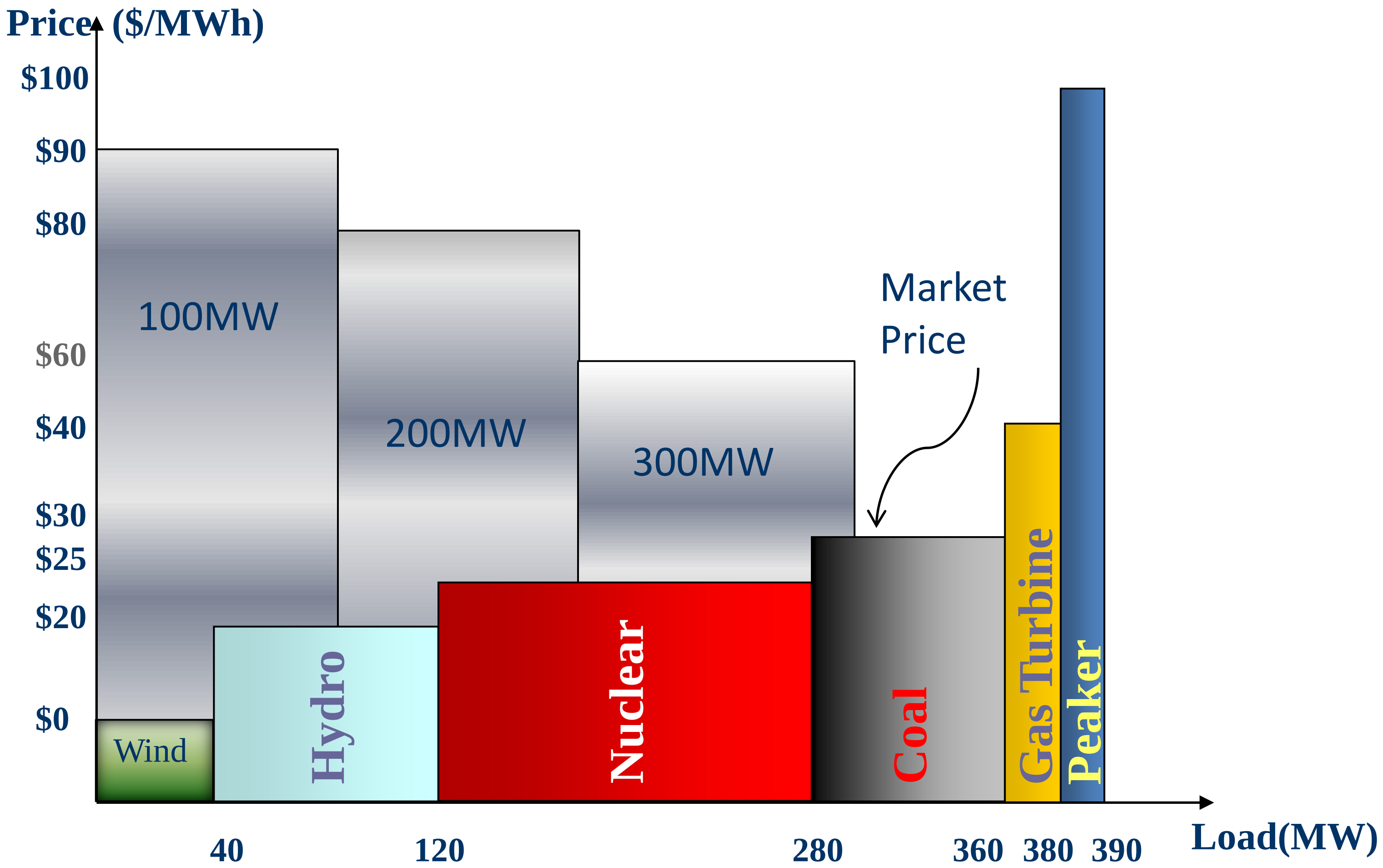
Why Is Electricity Different?

- Electricity cannot be stored
- Demand for electricity is inelastic
- Electricity produced must be dispatched
- Engineering system stability requires constant balance between demand and supply
- Electricity markets are regional e.g. Ontario prices different from NY prices

How does the Ontario market work?

- Power is traded at each of 24 hours per day.
- Generators offer power; users bid for power.
- Bids/Offers are prepared by 11PM the previous night for each hour but can be revised up until 4 hours ahead of the beginning of each hour.
- Each participant submits one or more ordered pairs into the market for that hour – (amount bid, price bid) or (amount offered, price offered).

Stack-Based Pricing



Bid/Ask strategy

- If you "have to have it" you bid a very high amount.   For instance GM – cost of power is tiny compared to cost of running assembly line.
- If you "have to sell it" you bid a very low, often a negative, amount. For instance Nuclear Power Plant.
- Price is usually set by the flexible people.

Special players

- Solar power generators are guaranteed \$443/MWh for all power they sell; wind and special green microhydro \$140/MWh.
- All of these power are bid in at -\$2000 to guarantee it is taken.

Ontario Electricity Market

- Unique dataset obtained from Environment Canada and Independent Electricity System Operator (IESO)
- A whole year (May 2011 – May 2012) hourly data containing:
  - ➢HOEP (Hourly Ontario Electricity Price, $/MWh)
  - ➢Market Demands (MW)
  - ➢Generators capabilities and outputs (MW)
  - ➢Temperature, wind speed, humidity and etc.

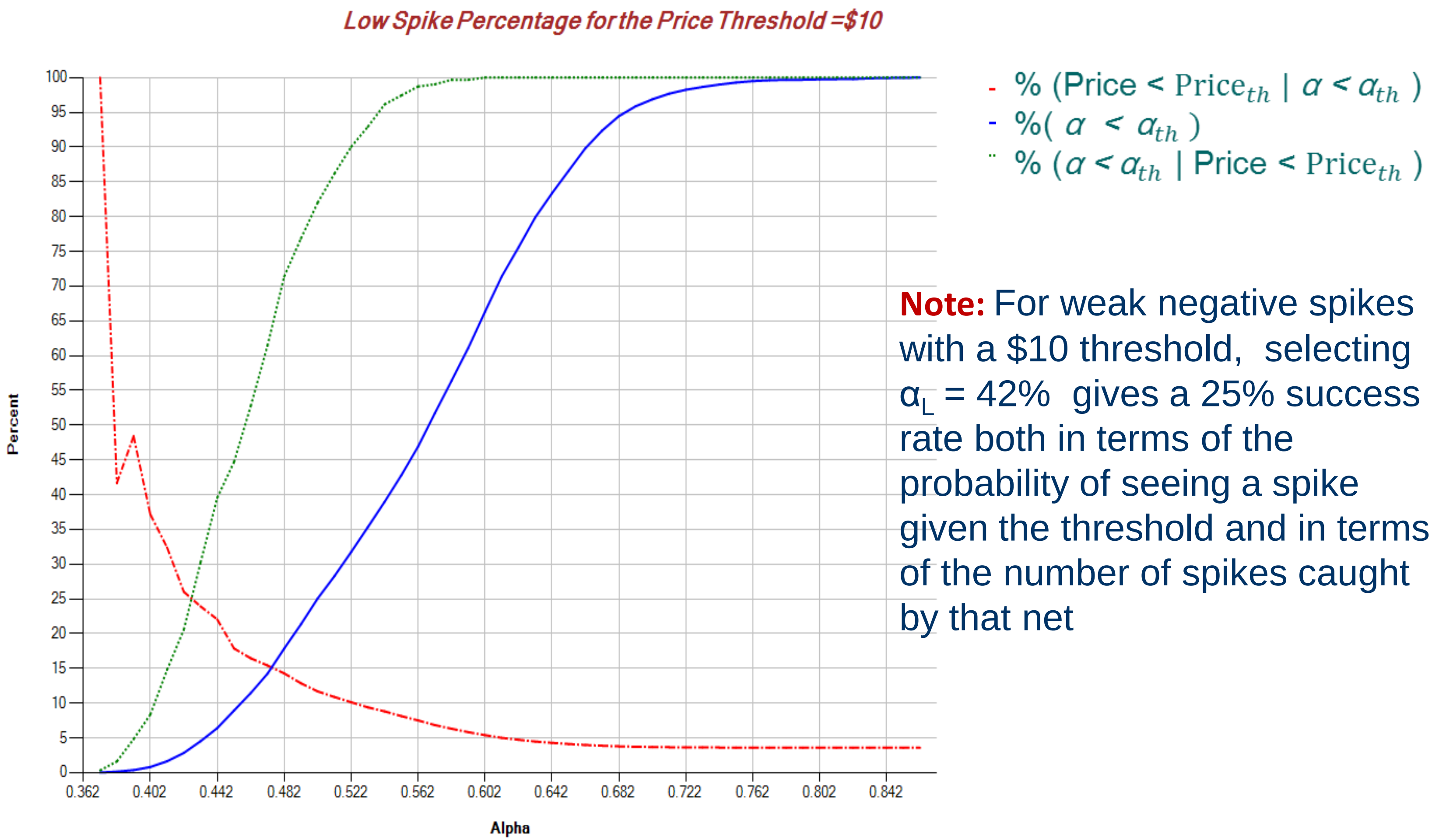
Three-Regime Switching Model

- Inspired by M. Davison, L. Anderson et al.(2002)
- Switching variables control the process
- Switching variable ratio of load to available generation

Negative price spikes

- New phenomenon: negative spikes
  - Associated with low demand states
  - Associated with high wind states
  - Associated with unexpected wind coming into the market when other units can’t be ramped down
  - (Test will be look at nuke output, look at rate of change).

Low price Percentage for the price threshold =10



Note: For weak negative spikes with a $10 threshold, selecting α_L = 42% gives a 25% success rate both in terms of the probability of seeing a spike given the threshold and in terms of the number of spikes caught by that net

The Wind α-Ratio and its relationship with price

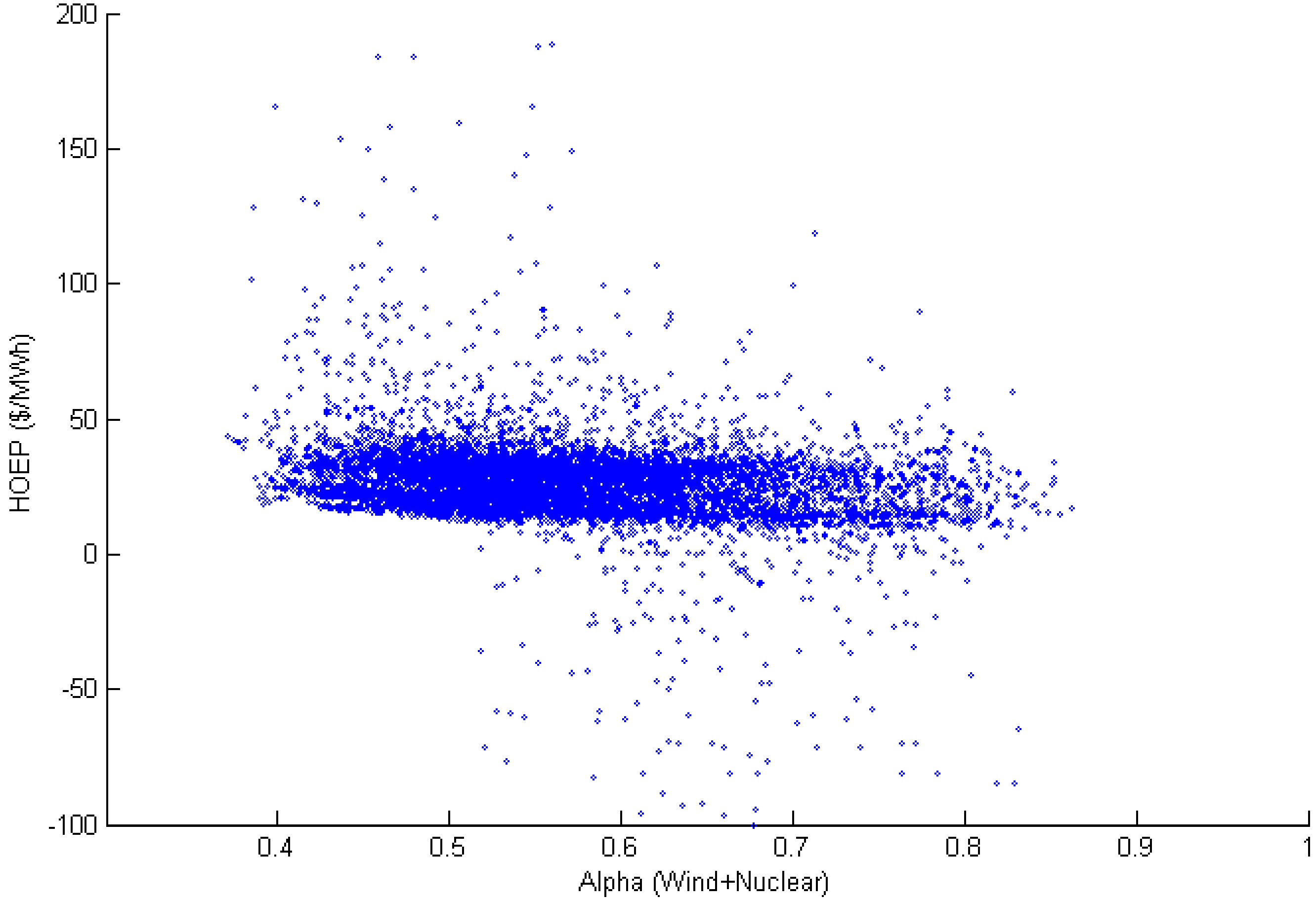
- Another primary driver of the switching variable is

$$\propto_{Wind}(t) = \frac{Wind\ Output(t)}{Ontario\ Load(t)}$$

- The following should be true:

$$Pr(price\ spike) \uparrow when\ \propto_{Wind}\downarrow$$
$$Pr(Negative\ price) \uparrow when\ \propto_{Wind}\uparrow$$

Wind + Nuclear alpha vs. HOEP



Some Conclusions from Data

- Some weak evidence to support the folklore that wind physics + nuke physics + regulation → negative prices.

Is the solution storage?

- Is energy storage the solution?
- Storage could buffer uncertainties. (Castrunuovo & Lopes 2004)
- See recent case study (Connolly et al 2012)
- But storage is expensive!

Batteries are expensive!

- A Nickel-Cadmium AA battery contains 1.2W-h of electricity, weighs about 30g and costs about $0.50.
- To store a Megawatt hour in AA batteries would take about 800,000 of them, for a total weight of 24,000 kg and cost about $400K.
- Even industrial size batteries are expensive as we'll see later.

Who pays for storage?

- In current Ontario setting, not the wind or solar producers
- The cost of uncertainty is another cost of running green markets.
- In fact, in Ontario if you did build a storage facility you'd have to buy and sell at the open market price: No Feed In Tariff

Existing work on Energy Storage

- Using detailed stochastic processes for prices (Carmona & Ludkovski 2008, 2010; Chen & Forsyth 2007)
- Using detailed engineering and stochastic processes (Thompson, MD, Rasmussen 2004, 2009; Chen & MD 2009a, 2009b)

Buffering Wind Energy

- Optimal sizing of PSR facilities: DeCesaro et al. 2009, Abbey & Joos 2009
- Optimal PSR facility operation: Castrunuovo & Lopes 2004, Garcia-Gonzalez et al 2008.
- Optimal use of large hydro reservoir to buffer wind: Denault et al. 2009
- Use of wind forecasts: Xie et al 2012
- Kim & Powell solve infinite horizon continuous time model incorporating much detail.

Impact of regulation on storage

- Today I present a very simple model of a storage facility with just 4 parameters:
  - the value of a unit of power,
  - the fractional loss in storing the power,
  - the probability of generating the power,
  - and the penalty from bidding power into the market that isn't delivered.
- The model generates a nonlinear system of difference equations that can be solved in closed form!
- This closed form solution allows us to obtain many insights.

Wind meteorology

- A wind producer produces \$M of electricity if it is windy, otherwise nothing.
- Each period it is windy with probability p and calm with probability 1-p, $0 \le p \le 1$.
- No access to forecasts, although many of the results shown here also hold for deterministic but time varying p_k.

Market Rules

- The wind producer must decide in advance whether to offer power into the market.
- If power is offered and it is windy, producer gets $M.
- If power is not offered it cannot be sold, whether or not it is windy.

Penalties

- If the producer offers power and can’t deliver must pay a penalty of $-xM$; $x \geq 0$
- Ontario market: $x = 0$;
- New York market: $x > 0$.

Storage physics

- The wind producer has access to a storage facility allowing them to store a single unit of wind energy. This storage can be filled or withdrawn in a single hour.
- (No claim that one storage unit is in any way optimal)
- Storage is "lossy" and we assess the cost of this loss at withdrawal. If a unit of energy is withdrawn it earns $(1-\gamma)M$, $0 \le \gamma \le 1$.

Storage contracts

- Storage facilities are leased for N periods.
- At the end of the lease, storage returned to its owner.
- If full, facility gets a cash refund of $(1-\gamma)M$,
- Return empty facility: get nothing.

V(F,k) and V(E,k)

- The value of a full storage facility, assuming optimal operation, with k periods left before the end of the lease is denoted by V(F,k)
- The value of an empty storage facility, assuming optimal operation, with k periods left before the end of the lease is V(E,k)

V(F,B,k) and V(F,N,k)

- With k periods remaining we must decide whether to offer power or not.
- The value of a full facility with k periods remaining given we offer is V(F,B,k)
- If we don't offer power the value of the full facility with k periods remaining is V(F,N,k).
- V(F,k) = max[V(F,B,k),V(F,N,k)]

V(E,B,k) and V(E,N,k)

- The value of an empty facility with k periods remaining given that we offer is V(E,B,k)
- If we don't offer power the value of the full facility with k periods remaining is V(E,N,k).
- V(E,k) = max[V(E,B,k),V(E,N,k)]

The recursion relation: empty

- We use dynamic programming to solve this.
- We've already 'turned around' time by describing everything in terms of time remaining.
- $V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$
- $V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$
- Since if you don't bid and it's empty, you might as well fill the facility to sell later.

The recursion relation: full

- $V(F,N,k) = pV(F,k-1) + (1-p)V(F,k-1) = V(F,k-1)$.
- The V(F,B,k) case is a bit harder because if we bid and it's not windy we can choose whether to pay the penalty or empty the storage. Hence:
- $V(F,B,k) = p[M + V(F,k-1)]$

  $+(1-p)\max[-xM+V(F,k-1), (1-\gamma)M+V(E,k-1)]$

A note on expectations

- It probably makes sense to optimize the expected value of the cash flows as done above since the procedure will be repeated many times.
- If, however, you want to add risk aversion via for instance a utility, that will only have the effect of distorting the probability, so replace p by q and the structure of the equations remains.

System 0

$V(F,B,k) = p[M+V(F,k-1)]+(1-p)\max[-xM+V(F,k-1), (1-\gamma)M+V(E,k-1)]$

$V(F,N,k) = V(F,k-1)$

$V(F,k) = \max[V(F,B,k),V(F,N,k)]$

$V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$

$V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$

$V(E,k) = \max[V(E,B,k),V(E,N,k)]$

$V(F,0) = (1-\gamma)M$

$V(E,0) = 0.$

Solving this

- Solution of this system requires at each time:
- **Optimal bidding rules**, when empty and when full, before we know if the wind will blow or not.
- The **optimal decision** about whether to pay the **penalty or empty** the storage in the full,bid,no wind case.
- Expressions for **V(F,k) and V(E,k).**

Analytic solution

- We find a complete analytic solution for this nonlinear system of difference equations.
- We could easily code this system
- Analytic solutions still are nice.
- I will now describe the proof methodology
- Draw insights about wind storage from it.

Theorem 1: Solving for V(F,k)-V(E,k)

Theorem 1:

For the above system of difference equations,

$V(F,k)-V(E,k) = (1-\gamma)M + \min[p\gamma M, (1-p^k)xM]$

Prove using Lemma 2:

Lemma 2:

Let $V(F,k) - V(E,k) = Mm(k) + (1-\gamma)M$. Then

$$m(k) = m(k-1) + \min[x(1-p), \gamma p - pm(k-1)] - \min[p, x(1-p), (1-p)m(k-1)];$$

$m(0) = 0$.

Proof: Insert above ansatz into System 0, use tedious but simple algebra and relations like $\max(a+b,a+c) = a + \max(b,c)$ to close.

Proof of Theorem 1:

Theorem 1 <-> $m(k) = \min[\gamma p, \ x(1-p^k)]$ solves the Lemma 2 system:

$$m(k) = m(k-1) + \min[x(1-p), \gamma p - pm(k-1)] - \min[p, x(1-p), (1-p)m(k-1)]; \quad m(0) = 0.$$

Proof: divide into 4 cases and use induction:

i) $x \geq \gamma * \max(1, p/(1-p)$

ii) $\gamma \leq x < \gamma \max(1, p/(1-p)$ ($\rightarrow p > ½$)

iii) $\gamma p \leq x < \gamma$

iv) $0 \leq x < \gamma p$

Discussion

- $V(F,k)-V(E,k) = (1-\gamma)M + \min[p\gamma M, (1-p^k)xM]$
- Nondecreasing in time remaining like an option.
- If $x \leq \gamma p$, $V(F,k)-V(E,k)$ is always increasing with a limit of $(1-\gamma+x)M$,
- If $x > \gamma p$, $V(F,k)-V(E,k)$ increases until $k = k^*$ and then reaches a limit of $[1-\gamma(1-p)]M$ at the finite time k^*.
- k^* is the largest integer satisfying

$$k^* < [\ln(x-\gamma p)-\ln(x)]/\ln(p)$$

The optimal control

The Theorem 1 “backbone” allows all other results to come easily.

Corollary 3:$-xM + V(F,k) \leq (1-\gamma)M+V(E,k)$ for all k.

Proof: $-xM \leq (1-\gamma)M - [V(F,k) - V(E,k)]$

$-xM \leq - \min[p\gamma M, (1-p^k)xM]$, (Thm 1) or

$x \geq \min[\gamma p, x(1-p^k)]$ which is clear.

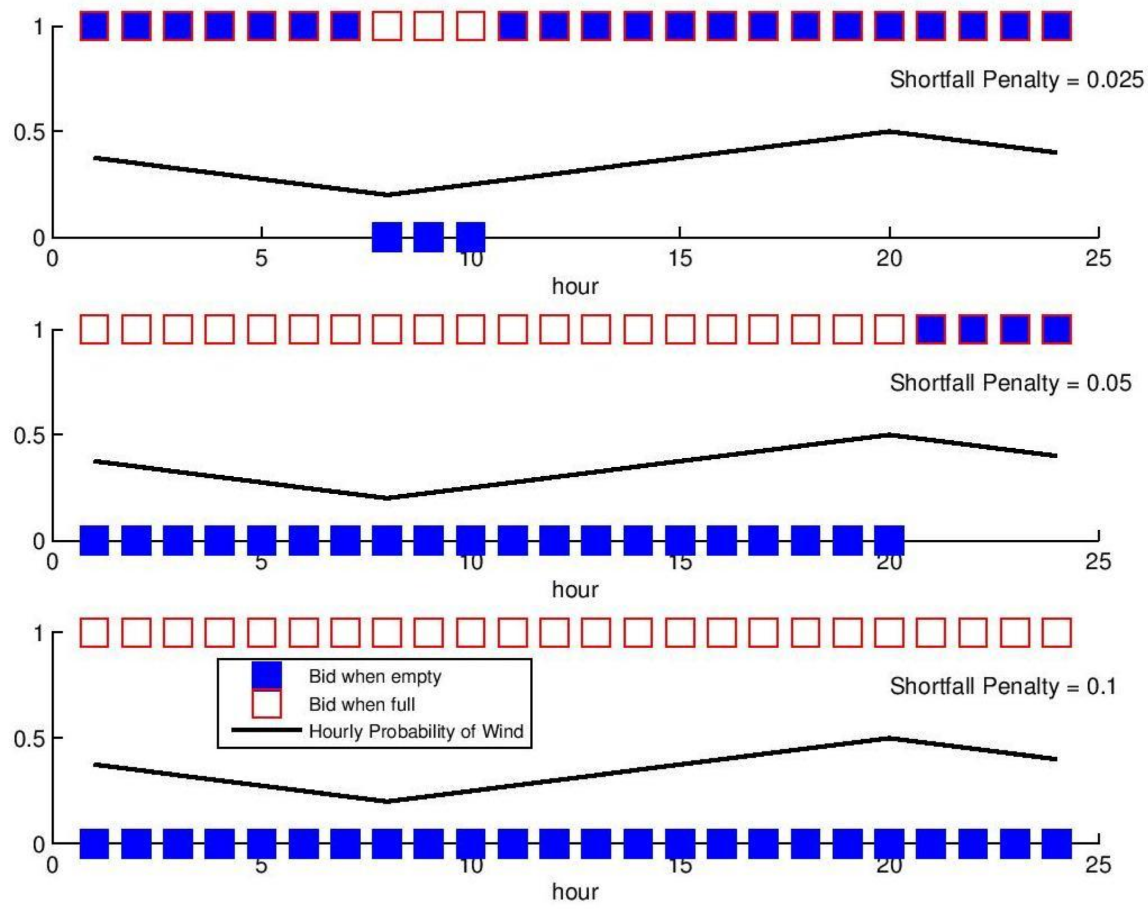
Never pay penalty if storage full

- Corollary 3 implies that, if you bid and the wind doesn’t blow, it is never optimal to pay the penalty but always better to empty the full storage. This is true even for tiny penalties, or for no penalties at all.
- (So we can simplify System 0 a bit by replacing the expression for V(F,B,k) with: $V(F,B,k) = p[M+V(F,k-1)]+(1-p)[(1-\gamma)M +V(E,k-1)$

Optimal offering rules

- For p, x, γ constant, all rules in closed form.
- The result is simple:
- Always offer power when you are full.
- Offer power when empty depending on the relationship between p, x, γ
- High x: don't offer when empty; low: never offer; medium – sometimes offer
- Next graphic gives intuition: time varying p.

Optimal bidding, variable wind p



Should always bid when full

Corollary 4: V(F,B,k) >= V(F,N,k) for all k.

Proof: From the above slide and System 0:

V(F,B,k)-V(F,N,k) = p[M+V(F,k-1)]+(1-p)[(1-γ)M
+V(E,k-1) – V(F,k-1)]
= pM + (1-p)){(1-γ)M – [V(F,k-1) – V(E,k-1)]}

Using Theorem 1, = pM – (1-p)min[pγM,x(1-p^{k-1})M]

- (1-p)M*max[p/(1-p) - pγ,-x(1-p^{k-1})]
  = M*max[p(1-γ(1-p), -x(1-p) (1-p^{k-1}) >= 0,
  since γ*(1-p) < 1.

Bidding rules when empty

$V(E,B,k) – V(E,Nk)$ = $p[M+V(E,k-1)]$ + $(1-p)[-xM+V(E,k-1)] – \{pV(F,k-1) + (1-p)V(E,k-1)\}$

$= pM – p[V(F,k-1)-V(E,k-1)] – (1-p)xM$

$= [p-(1-p)x]M – p[(1-\gamma)M + \min\{\gamma p,x(1-p^{k-1})\}M]$

$= [\gamma p – (1-p)x]M - pM*\min\{\gamma p,x(1-p^{k-1})\}$

$= [\gamma p – (1-p)x]M + pM*\max\{-\gamma p,-x(1-p^{k-1})\}$

$= \max\{\gamma p – (1-p)x, \gamma p - x(1-p^k)\}M$. But, unless $k = 0$ in which case we can't bid anyway, $1-p \geq 1-p^k$,

Hence $V(E,B,k) – V(E,N,k) = [\gamma p - x(1-p^k)]M$

Empty bid rules: Large penalties

- $V(E,B,k) – V(E,N,k) = [\gamma p - x(1-p^k)]M$
- Large penalty: $x >= \gamma * \max[1,p/(1-p)]$
- Then, if $p < ½$, $x(1-p^k) > x(1-p) > px > \gamma p$ and the expression is negative. If $p > ½$ $x(1-p^k) > x(1-p) > \gamma p$ and the expression is still negative.
- so $V(E,B,k) – V(E,N,k) < 0$
- and so it's optimal not to bid.

Optimal control: large penalties

- If $x \geq \gamma^* \max[1,p/(1-p)]$ then the optimal control is to bid when full and not bid when empty. That way you never have to pay penalties and you refill the first time it's windy after a calm day.
- Note that sufficiently huge penalties are never collected!
- Here $V(F,k)-V(E,k) = [1-\gamma(1-p)]M$

Empty bid rules: Small penalties

- $V(E,B,k) – V(E,N,k) = [\gamma p - x(1-p^k)]M$
- Small penalty: $x \leq p\gamma$
- Then $x(1-p^k) < x \leq p\gamma$
- so $V(E,B,k) – V(E,N,k) > 0$
- and so it's always optimal to bid.
- Here $V(F,k)-V(E,k) = [1-\gamma p + x(1-p^k)]M$

Optimal control: small penalties

- With small penalties you always bid whether you are full or empty. The effect of this is that if you start empty you never fill the storage, and if you start full you only use the storage once, to empty it.
- So the penalties are too small to encourage use of the storage, even though it looks like you are using the storage when it's full.

Empty bid rules: medium penalties

- $V(E,B,k) – V(E,N,k) = [\gamma p - x(1-p^k)]M$
- Medium penalty: $\gamma p < x < \gamma \max[1,p/(1-p)]$
- Here $\gamma p - x(1-p^k)$ is positive (when $k < k^*$) or negative (when $k \geq k^*$).
- Here k^* is the largest integer satisfying
- $k^* < \ln[1-\gamma p/x]/\ln(p)$
- So you don't bid (sufficiently far from maturity) and then bid (sufficiently close to maturity)

Optimal control: medium penalties

- With sufficiently small time remaining, you might be able to "get away" with bidding even when empty, in the expectation of never having to pay a penalty.
- $x(1-p^k)$ is the expected proportional penalty paid with k time steps remaining.
- Eventually it's better to play it safe and bid, with proportional loss of γ incurred with probability p.
- Hence we compare γp and $x(1-p^k)$.

Facility values: large penalties

- Here the equations are:
  $V(F,k) = V(F,B,k)$
  $= p[M+V(F,k-1)]+(1-p)[\ (1-\gamma)M+V(E,k-1)]$
- $V(E,k) = V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$
- So $V(E,k) = V(E,k-1) + p[V(F,k-1)-V(E,k-1)]$
- $V(E,0) = 0$ and $V(F,k)-V(E,k) = [1-\gamma(1-p)]M$, so
- **$V(E,k) = kp[1-\gamma(1-p)]M \quad (k \geq 0)$.**
- **$V(F,k) = (kp+1)[1-\gamma(1-p)]M \ (k \geq 1)$.**

Facility Values: Small penalties

- Here V(F,k) = V(F,B,k) and V(E,k) = V(E,B,k) so the recursion relations are:
- $V(F,k) = p[M+V(F,k-1)]+(1-p)[(1-\gamma)M+V(E,k-1)]$
- $V(E,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$ or
- $V(E,k) = V(E,k-1) + [p-x(1-p)]M$
- Or $\mathbf{V(E,k) = k[p-x(1-p)]M}$
- $\mathbf{V(F,k) = k[p-x(1-p)]M + (1-\gamma)M + x(1-p^k)M.}$

Facility Values: Medium penalties

- When $k < k^*$ it's as if the penalties were small, so $V(E,k) = k[p-x(1-p)]M, \; k < k^*$
- When $k > k^*$ the penalties are now large, so we can solve the large penalty difference equation with the "initial condition" $V(E,k^*) = k^*[p-x(1-p)]M$
- $V(E,k) = V(E,k-1) + p[V(F,k-1)-V(E,k-1)]$, where for $k > k^*$, $V(F,k-1)-V(E,k-1) = [1-\gamma(1-p)]M$, so
- **$V(E,k) = kpM - k^*x(1-p)M - (k-k^*)\,\gamma(1-p)M, \; k \geq k^*$.**
- **k^*: largest int satisfying $k^* < \ln[1-\gamma p/x]/\ln(p)$**

Impact on storage values

- So far the analysis has only told us what to do if we were given a storage facility.
- In this light it’s not so surprising that we’d choose to empty a full facility rather than pay penalties.
- But what if we had to rent a storage facility? Would it be worth it?
- We need to compare with a turbine operated without a companion storage.

Compare with turbine with no storage

- Consider a turbine with no storage. W(k) is the value of this turbine with k periods remaining.
- W(k) follows the difference equation:
- W(k) = max[W(B,k), W(N,k)]
- W(B,k) = p[M + W(k-1)] + (1-p)[-xM + W(k-1)]
- W(N,K)= pW(k-1) + (1-p)W(k-1) = W(k-1)
- So W(k)=W(k-1)+max[p-x(1-p),0]*M; W(0) = 0.
- So **W(k) = kM*max[p-x(1-p),0].**

No storage wind turbine: controls

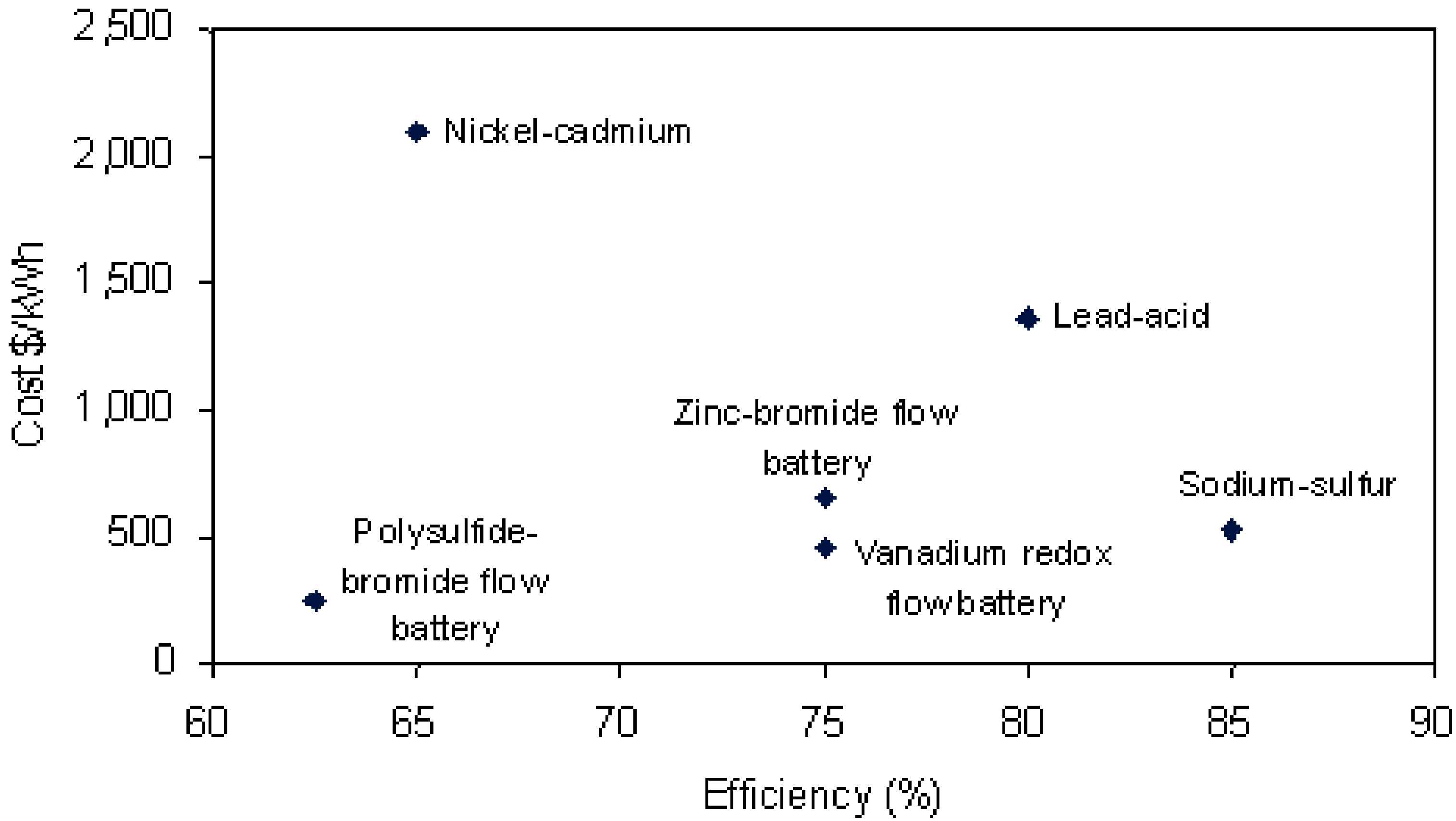
- If $x < p/(1-p)$ the fines are small enough to make it worthwhile to operate, and you will always bid, and have $W(k) = kM^*[p-x(1-p)]$.
- If $x > p/(1-p)$ the fines are large enough for the best policy be never to bid, with $W(k) = 0$.
- These are the correct “comparator” values for the combined wind– storage facility.

Added value of high penalty storage

- Here base case is to have no value from wind, so penalty is equivalent to a law requiring wind turbine operators to operate storage (or other backup) facility.
- The added value from the storage, for a N period facility, is $Np[1-\gamma(1-p)]M$
- Note this value doesn't depend on the value of x (once it's big enough).
- It says that the more wind the better, the longer the facility life the better, and the more efficient the facility the better.

Battery costs vs. efficiencies

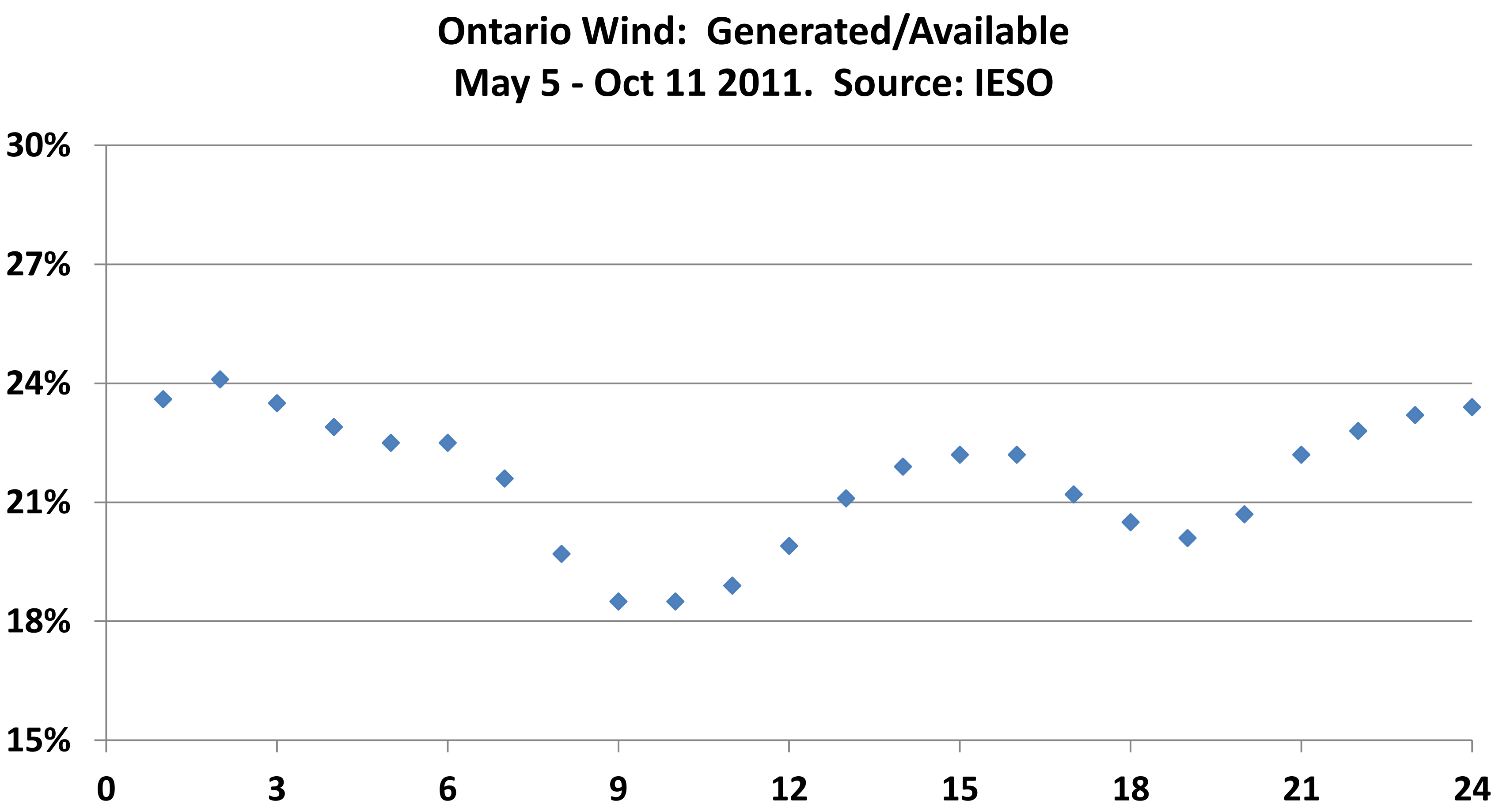
(Source: The Future of Energy Storage, Global Business Insights)



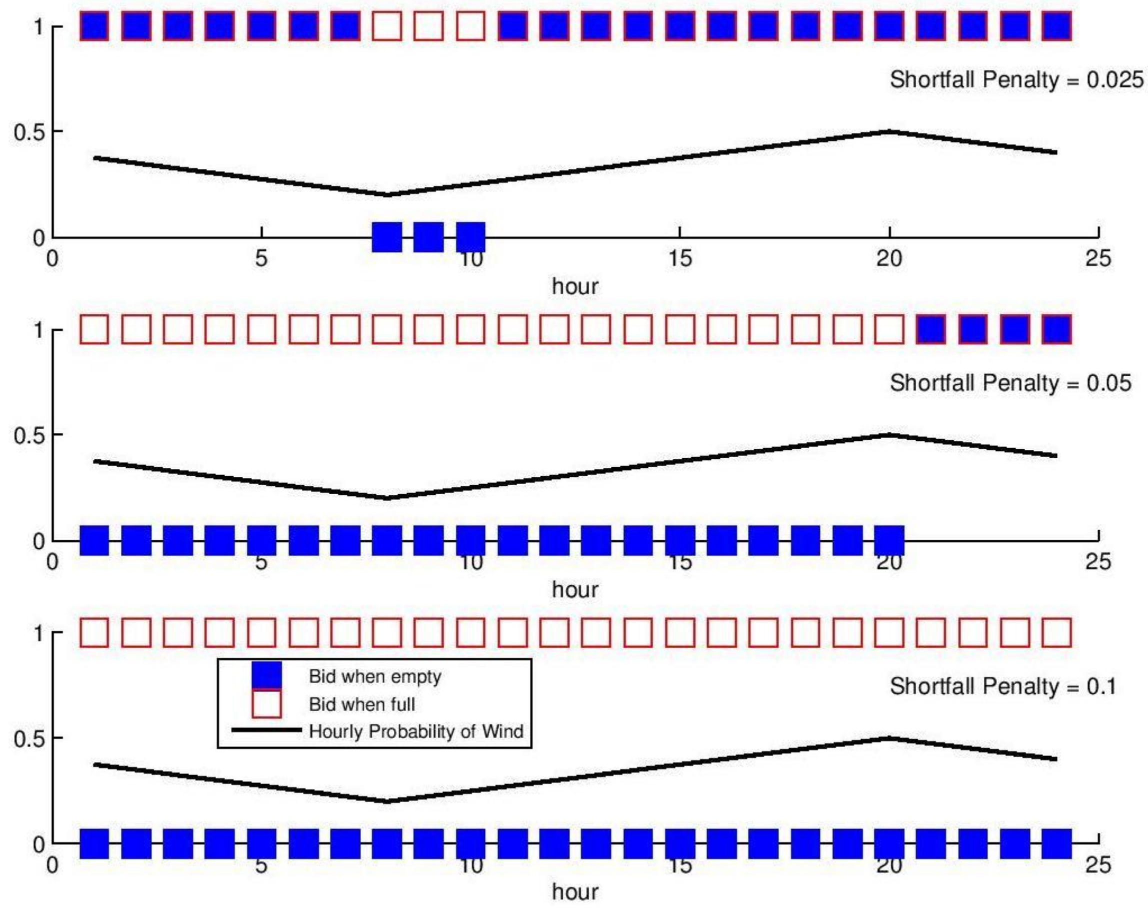
Estimating p

- We have access to the total production of wind in Ontario at each time and to the total availability of wind turbines at each time.
- If our model were correct for each turbine, we'd expect the long run average of this ratio to be the probability of full output.
- Next slide shows the data
- Yields (slightly conservative) estimate p = 20%

Estimating p (data)



Optimal bidding, variable wind p



Value: High penalty cost

- Take $M = \$140$ (i.e. 1MW turbine).
- Take $\gamma = 15\%$ (Sodium-Sulfur battery)
- Take $p = 20\%$.
- Take $N = 8760$ hours
- Then the value of the storage is about \$215,000 per MWh (per year).
- Cost is about \$500,000 but lasts for a number of years. So storage is "in the conversation".

Added value of low penalty storage

- Here $x < \gamma p$, so $x < p$ so $x < p/(1-p)$ and the base case is the "always run" no storage facility with value $W(k) = kM^*[p-x(1-p)]$.
- Here, though $V(E,k) = k[p-x(1-p)]M$, additional value of the storage is zero (since it doesn't change bidding behaviour).
- On the other hand, in this regime we'd expect the regulator to collect on average $N^*(1-p)xM$ in penalties, which could be used to defray the costs of storage. With N = 8760, p = 20%, M = $140 and $x = \gamma p$ = 3%, this is about $30,000.

Conclusions

- A simple model can be exactly solved and shows some interesting intuition.
- Of course this is way too unrealistic for reality – we need correlated wind speeds, storage with ability to store fractional units and seasonality, at the very least!
- Our key focus now is adding simple weather forecast models to this.
- It's fun to see how much insight we can get without a lot of computing.