Commodity price modeling in EDF. Parameter estimation and calibration

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Agenda

Commodity modeling in EDF: objectives

- Short term (1 day $\rightarrow$ 2 weeks) prediction
- Market comprehension
- Mid term risk management
  - Gross energy margin prediction
  - Risk measurement
  - Hedging
- Pricing
  - Valuation of Production assets
  - Valuation of flexibilities in supply contracts
- Investment decision

EDF portfolios: some examples

- **Production units**
  - Approximation : Strip of European spread options of payoff $CF(t)$

$$CF(t) = \left(S_t^{power} - hS_t^{fuel} - h'S_t^{CO2} - K\right)^+ \qquad \forall t \in [0\ ;\ T]$$

  - More realistic: dynamic constraints
    - Startup costs, limited number of startups
    - Scheduled outage periods
- **Gas storage, Hydro dam**
  - Swing option : lets the holder buy a flexible quantity $Q \in [Q_{min}\ ;\ Q_{max}]$
  - Additional constraints: minimal and maximal quantity per day, per month...
- **Supply contracts**
  - Indexed price: moving average opions
  - Flexibility in quantity: swing options

Spot market

- Daily spot for fuels
- (semi)Hourly spot price for power

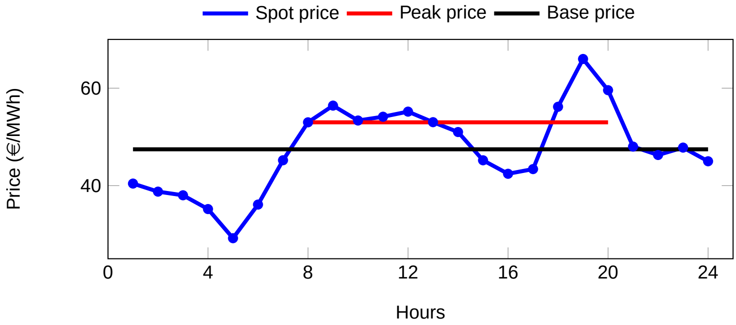
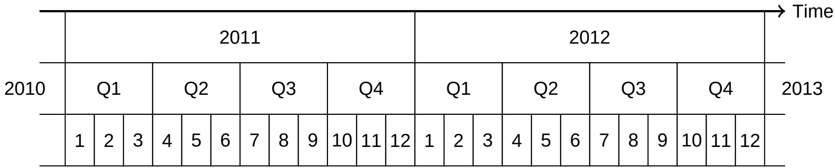


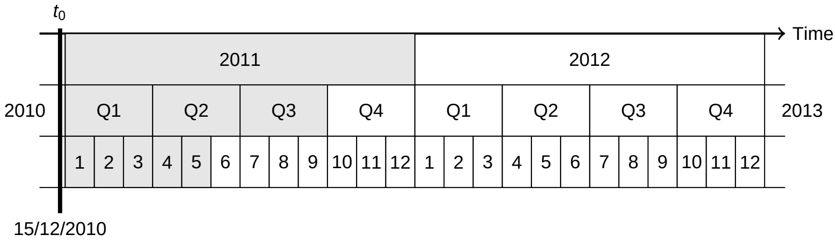
Figure : French power spot prices on January 12th 2012

- **Power specificity: delivery period**
- Different maturities and different delivery periods



Future market

- **Power specificity: delivery period**
- Different maturities and different delivery periods



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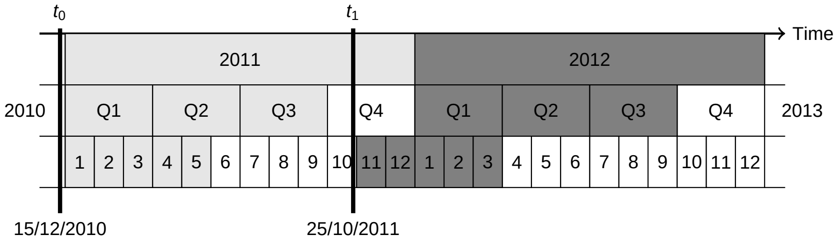

Future market

- **Power specificity: delivery period**
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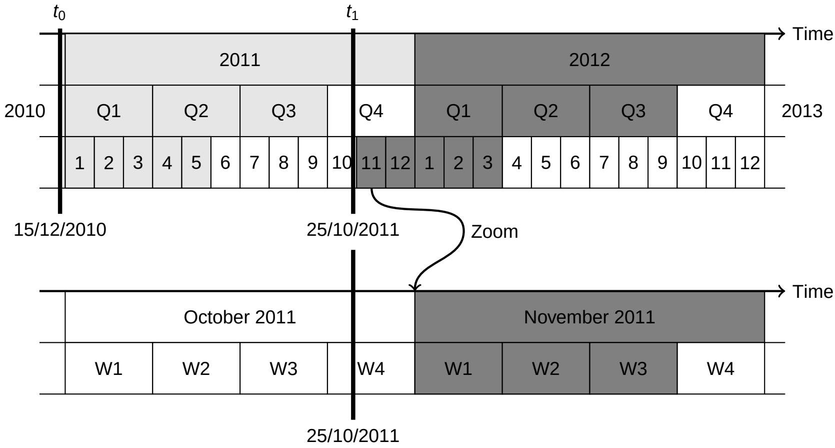

Contraints of modeling

- **Portfoltio exposition**
  - Power and fuels spot prices (or "unitary forward prices")
  - Power and fuels forward products
  - Link between power and fuels
- **Available hedging products**
  - forward products (with delivery periods for power)
- **Incomplete market: No derivatives (market information) on**
  - "unitary" (instantaneous) forward prices
  - any link (spread) between fuels and power
- **Consequences**
  - We need to model spot prices and forward products
  - The link is obtained from the forward curve
- **Calibration issues**
  - 2 different sets of observed data (spot prices and forward products)
  - Complex relationship between parameters and observed forward products

Electricity price modeling: two approaches

- **Forward curve of unitary future price as a starting point**

$$dF_t(T) = \mu(t, T, F_t(T))dt + \sigma(t, T, F_t(T))dW_t$$

  - Spot price deduced as a limit $S_t = \lim_{T \to t} F_t(T)$
  - Forward products deduced by no-arbitrage principles

$$F_t(T, \theta) = \int_0^\theta w(u) F_t(T + u) du$$

- **Spot price as a starting point**

$$dS_t = \mu(t, S_t, X_t)dt + \sigma(t, S_t, X_t)dW_t$$

  - Some other stochastic processes X_t (fuel prices, demand...) may turn up.
  - Forward prices deduced by no-arbitrage principles

$$F_t(T) = \mathbb{E}_t^{\mathbb{Q}}[S_T]$$

Two different works

- **Parameter estimation of the model currently used in EDF for risk management**
  - 2-factor model description
  - Estimation / Calibration on forward products and spot prices
  - Impact of calibration method on European option valuation
- **Structural model for commodity prices**
  - Model description
  - Study on the forward products reconstruction
  - Integration of parameters uncertainty in the forward products recontruction
  - Calibration
- **Interests: performance of a model and its calibration**
  - in representing market information
  - in introducing information

Two different works

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Two-factor model description

- **Formulation**

$$\frac{dF_t(T)}{F_t(T)} = \sigma_s(t)e^{-\alpha(T-t)}dW_t^{(s)} + \sigma_l(t)dW_t^{(l)}$$

with $\alpha \in \mathbb{R}_*^+$ and $\sigma_s(t)$ and $\sigma_l(t)$ are positive integrable functions.

- **"Discretized" forward product formulation** $F_t(T,\theta) = \frac{1}{\theta}\sum_{i=0}^{\theta-1} F_t(T+i)$

- **Forward product process**

$$dF_t(T,\theta) = \frac{1}{\theta}\sum_{i=0}^{\theta-1}\left[\sigma_s(t)e^{-\alpha(T+i-t)}dW_t^{(s)} + \sigma_l(t)dW_t^{(l)}\right]F_t(T+i)$$

The forward product process is not Markovian [Benth & Koekebakker (2008)]

- ***shaping factors*** $\lambda_i^{t,T,\theta} = \frac{F_t(T+i)}{F_t(T,\theta)}$ and $\Psi(\alpha,t,T,\theta) = \sum_{i=0}^{\theta}\lambda_i^{t,T,\theta}e^{-\alpha i}$

$$\frac{dF_t(T,\theta)}{F_t(T,\theta)} = \sigma_s(t)e^{-\alpha(T-t)}\Psi(\alpha,t,T,\theta)dW_t^{(s)} + \sigma_l(t)dW_t^{(l)}$$

Approximation on shaping factors, Markovian process

- Strong approximation for the calibration
  - Constant shaping factors $\lambda_i^{t,T,\theta} \equiv 1$
  - Constant volatility functions $\sigma_s(t) \equiv \sigma_s$ and $\sigma_l(t) \equiv \sigma_l$ [Kiesel et al. (2007)]

- Consequences: $\Psi(\alpha, t, T, \theta) \equiv \Psi(\alpha, \theta)$ is determistic and the dynamics on forward products becomes Markovian

- Two different methods of parameters estimation
  - Estimation on *Marginal volatilities* (average of log-returns variance over the cotation period)
    - Express the theoretical marginal volatility of a forward product as a function of model parameters
    - Fit the theoretical marginal volatility to the empirical (or implied) marginal volatility computed from observed cotations
  - Estimation from spot prices
    - Long term volatility estimated from long term forward products
    - Short term parameters estimated from spot prices

Estimation on marginal volatilities

- N observations $(F_n(T,\theta))_{n=1,\dots,N}$ at dates $(t_n)_{n=1,\dots,N}$, with $t_{n+1} - t_n = \delta t$
- Model on geometric returns

$$R_n(T,\theta) = \ln\left(\frac{F_{n+1}(T,\theta)}{F_n(T,\theta)}\right) = \sigma_s e^{-\alpha(T-t_n)}\Psi(\alpha,\theta)\sqrt{\nu(\delta t)}\varepsilon_n^s + \sigma_l\sqrt{\delta t}\varepsilon_n^l$$

with $\nu(\delta t) = \frac{e^{2\alpha\delta t}-1}{2\alpha}$

- Marginal volatility: average of instantaneous return variance

$$MV^2(T,\theta,\alpha,\sigma_s,\sigma_l) = \frac{1}{N}\sum_{n=1}^{N} Var\left[R_n(T,\theta)\right]$$

- Historical (or implied) marginal volatility $\hat{MV}^2(T,\theta)$ from observed forward returns (or options)
- Estimation: minimization of the squared difference

$$(\hat{\alpha},\hat{\sigma_s},\hat{\sigma_l}) = \underset{(\alpha,\sigma_s,\sigma_l)}{\arg\min}\sum_{(T,\theta)}\left(\hat{MV}^2(T,\theta) - MV^2(T,\theta,\alpha,\sigma_s,\sigma_l)\right)^2$$

Two-factor model: Estimation on spot prices

$$\frac{dF_t(T)}{F_t(T)} = \sigma_s e^{-\alpha(T-t)} dW_t^{(s)} + \sigma_l dW_t^{(l)}$$

- Spot price formulation:

$$\ln S_t = \ln F_0(t) - \frac{1}{2}\left[\sigma_s^2 \frac{1-e^{-2\alpha t}}{2\alpha} + \sigma_l^2 t\right] + \underbrace{\int_0^t \sigma_s e^{-\alpha(t-u)} dW_u^{(s)}}_{X_t^{(s)}} + \underbrace{\int_0^t \sigma_l dW_u^{(l)}}_{X_t^{(l)}}$$

- Deseasonalization step
- Long term volatility σ_l estimated from long term forward products
- Short term parameters estimated from a state-space model
  - State equation: two hidden factors $X_t^{(s)}$ and $X_t^{(l)}$
  - σ_s and α estimated by Likelihood maximization on deseasonalized spot prices, by using a Kalman filter

Estimation results

- Date of estimation: 12-Mar-2013
- 1 year of historical data
- Two different estimation methods
  - Estimation on (empirical) marginal volatilities
  - Estimation on long term forward products (for the long term factor) and spot prices (for the short term factor)
- Study on resulting parameter values: strip of European options on monthly forward products (Apr-2013 $\rightarrow$ Mar-2015)

$$(F(t,T,\theta) - K)^{+}, \qquad K = F(t_0,T,\theta)$$

  - Comparison in European option pricing
  - "Benchmark" value defined from a multi-factor model fitted on empirical volatilities $\hat{MV}^2(T,\theta)$

Results: estimation on UK Power

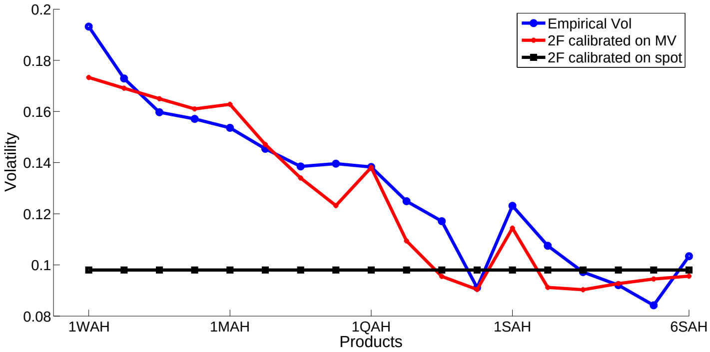

Figure : Calibration results on UK power

parameter	Estimation on MV	Estimation on spot
σ_S	19.1%	84.5%
α	1.37	162.65
σ_l	9.8%	9.8%

Results: European options pricing (UK Power)

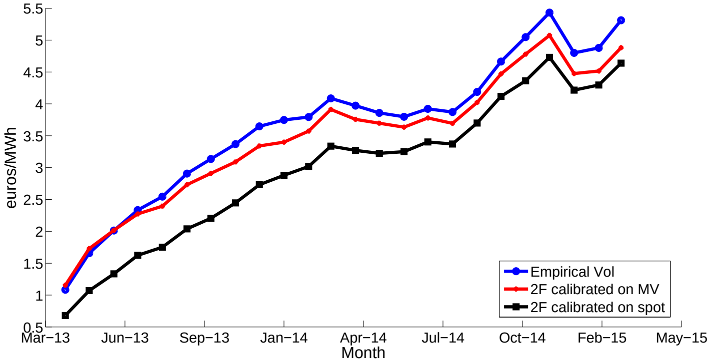

Figure : European option princing on UK Power

	Empirical volatility	Estimation on MV	Estimation on spot
MtM (€)	64 252	60 787	52 295
Relative difference	–	-5%	-19%

Conclusion

- **2 factor model objectives**
  - Represent spot prices and forward products
  - Can be calibrated on historical or implied volatility, and on spot prices
  - Can fit perfectly the forward products at a given date
- **Caibration issues:** need approximations
  - Constant and equally weighted shaping factors
  - Constant volatility function $\sigma_s(t)$ and $\sigma_l(t)$
  - No statistical results
  - Calibration methods have a high impact on indicators
    - A "benchmark" value can give e reference for forward derivatives
    - More difficult for options on spot prices
- **Some other issues**
  - No specific event (spikes, negative prices...)
  - Weak relationship between commodities

Two different works

- **Parameter estimation of the model currently used in EDF for risk management**
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Fundamental approach

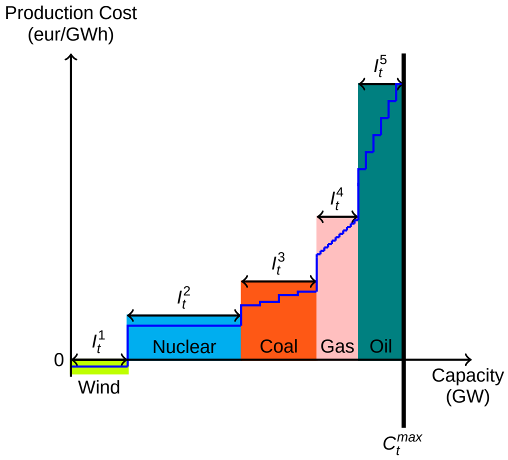
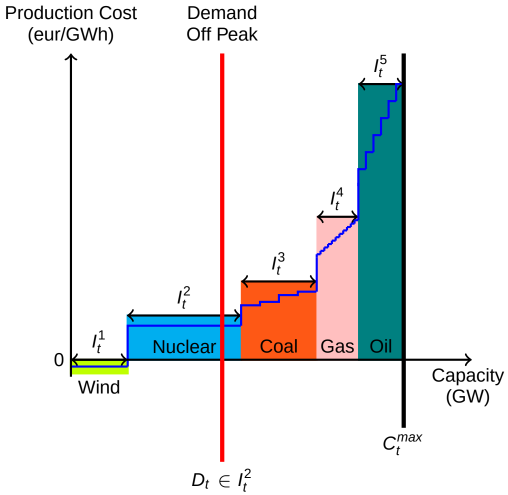


Figure : Structural model [Aïd et al. (2012)]

Fundamental approach



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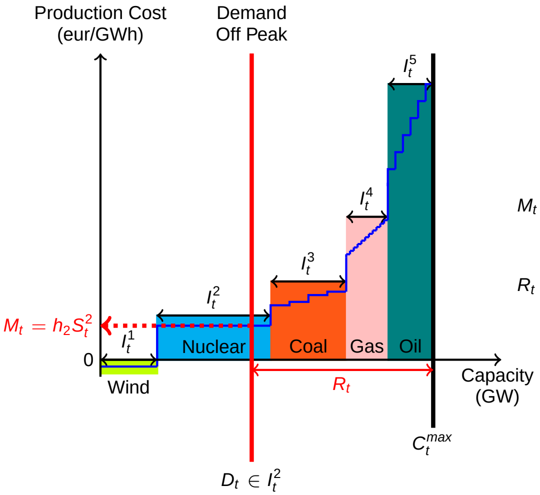
Fundamental approach

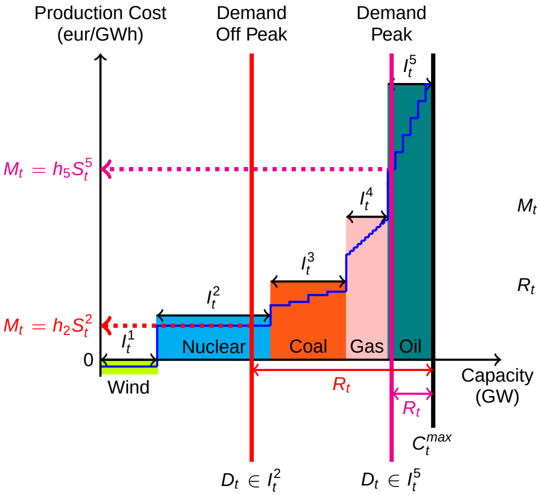


$$M_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{D_t \in I_t^i}$$

$$R_t = \sum_{i=1}^{n} C_t^i - D_t$$

Figure : Structural model [Aïd et al. (2012)]

Fundamental approach



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Fundamental approach

$$S_t = \frac{\gamma}{R_t^{\nu}} M_t$$

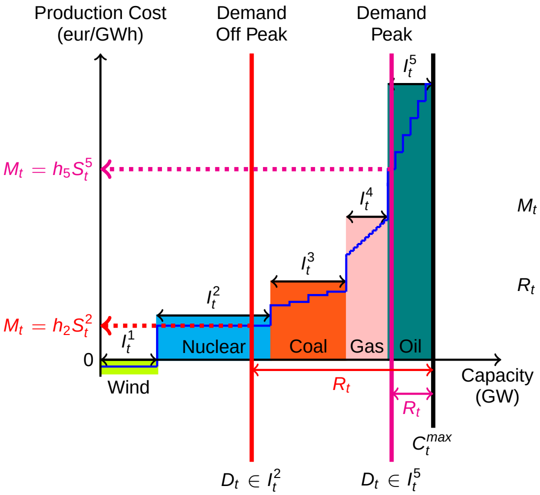

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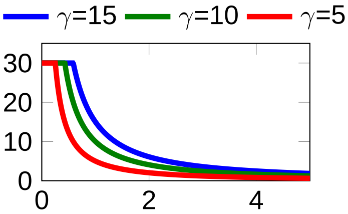
Figure : Structural model [Aïd et al. (2012)]

Power spot price modeling

$$S_t = g\left(C_t^{max} - D_t\right) \sum_{i=1}^{n} h_i S_t^i \mathbb{1}_{D_t \in I_t^i}$$

with

- D_t demand at t,
- C_t^i production capacity of fuel i,
- C_t^{max} total capacity,
- S_t^i spot price of fuel i,
- h_i heat rate of production with fuel i.
- I_t^i capacity interval where fuel i is marginal.
- $g(x) = \min\left(M, \frac{\gamma}{x^\nu}\right) \mathbb{1}_{x>0} + M \mathbb{1}_{x<0}$



Results on spot prices

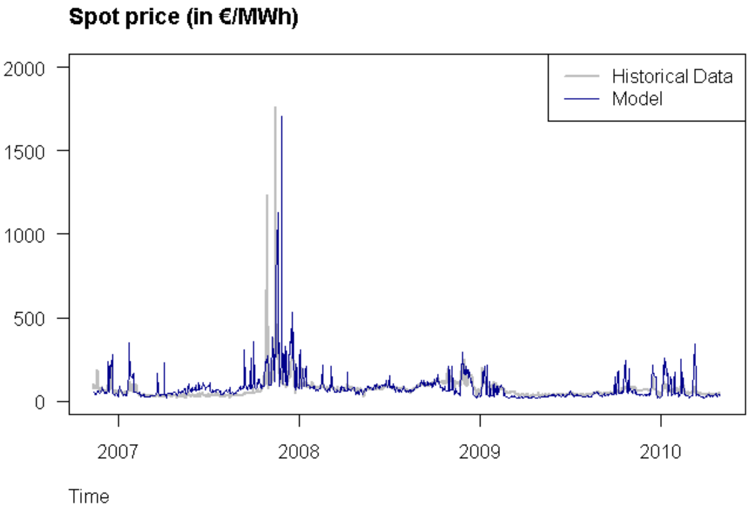


Figure : Spot price reconstruction from the structural model

Consequence on forward prices

- No-arbitrage condition

$$F_t(T) = \mathbb{E}^{\mathbb{Q}}[S_T | \mathcal{F}_t]$$

- Induced relation between forward prices

$$F_t(T) = \sum_{i=1}^{n} \underbrace{\mathbb{E}^* \left[g\left(C_T^{max} - D_T\right) \mathbf{1}_{D_T \in I_T^i} | \mathcal{F}_t \right]}_{G_T^i(t, C_t, D_t)} h_i F_t^i(T)$$

- Final reconstruction (assuming $F_t^i(T') = F_t^i(T, \theta)\ \forall T' \in [T\ ;\ T + \theta]$:

$$\boxed{F_t(T, \theta) = \sum_{i=1}^{n} \underbrace{\left(\frac{1}{\theta} \sum_{T'=T}^{T+\theta} G_{T'}^i(t, C_t, D_t) \right)}_{\text{stochastic weights}} h_i F_t^i(T, \theta)}$$

Modeling demand and capacities

Deterministic part + stochastic part (Ornstein-Uhlenbeck)

$$D_t = f_D(t) + Z_D(t)$$
$$C_t^i = f_i(t) + Z_i(t)$$

- Deterministic part:

$$f_D(t) = d_1^{(D)} + d_2^{(D)} \cos\left(2\pi(t - d_3^{(D)})\right) + \mathbf{week}_D(t)$$
$$f_i(t) = d_1^{(i)} + d_2^{(i)} \cos\left(2\pi(t - d_3^{(i)})\right) + \mathbf{week}_i(t)$$

- Stochastic part: Ornstein-Uhlenbeck process

$$dZ_D(t) = -\alpha_D Z_D(t)dt + \beta_D dW_t^D$$
$$dZ_i(t) = -\alpha_i Z_i(t)dt + \beta_i dW_t^i$$

Study on forward reconstruction

$$F(t,T,\theta) = \sum_{i=1}^{n}\left(\frac{1}{\theta}\sum_{T'=T}^{T+\theta} G^i_{T'}(t,C_t,D_t)\right)h_i F^i(t,T,\theta)$$

$$G^i_{T'}(t,C_t,D_t) = \mathbb{E}^*\left[g\left(C^{max}_T - D_T\right)\mathbf{1}_{D_T\in I^i_T}|\mathcal{F}_t\right]$$

- **Model specifications**
  - 3 types of capacities: nuclear, (gas/coal), oil
  - Carbon taken into account
  - Estimation of Demand and Capacities model parameters $d_1^{(*)}$, $d_2^{(*)}$, $d_3^{(*)}$ and $\mathbf{week}_*(t)$ on two years of historical data from RTE[1]
  - Observation of commodity forward products from Platts [2]

[1] www.rte-france.fr

[2] www.platts.fr

Electricity forward reconstruction: Month-ahead

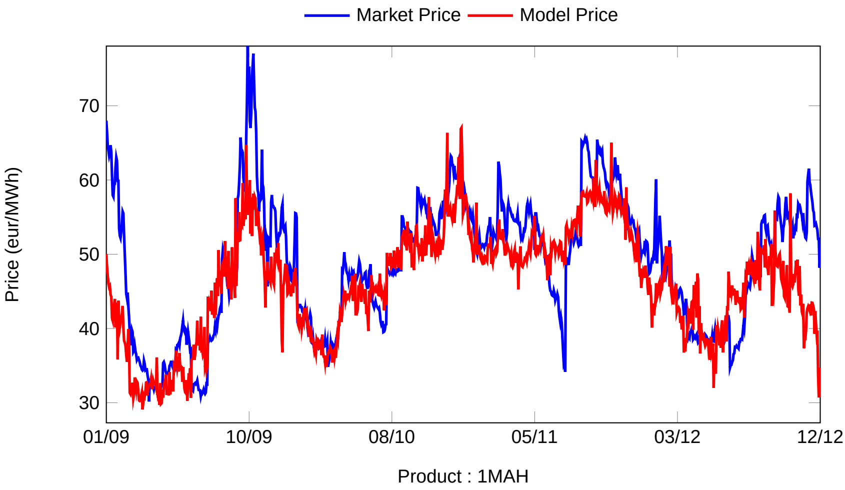


Figure : 1-Month-ahead reconstruction: observed prices (blue) and reconstructed forward prices from complete structural model with (red) and without (pink) the scarcity function

Electricity forward reconstruction: Quarter-ahead

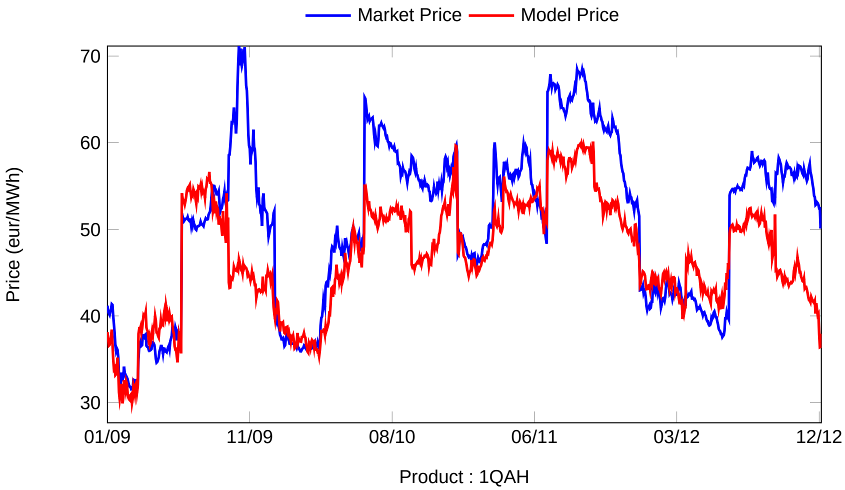


Figure : 1-Quarter-ahead reconstruction: observed prices (blue) and reconstructed forward prices from complete structural model with (red) and without (pink) the scarcity function

Electricity forward reconstruction: Year-ahead

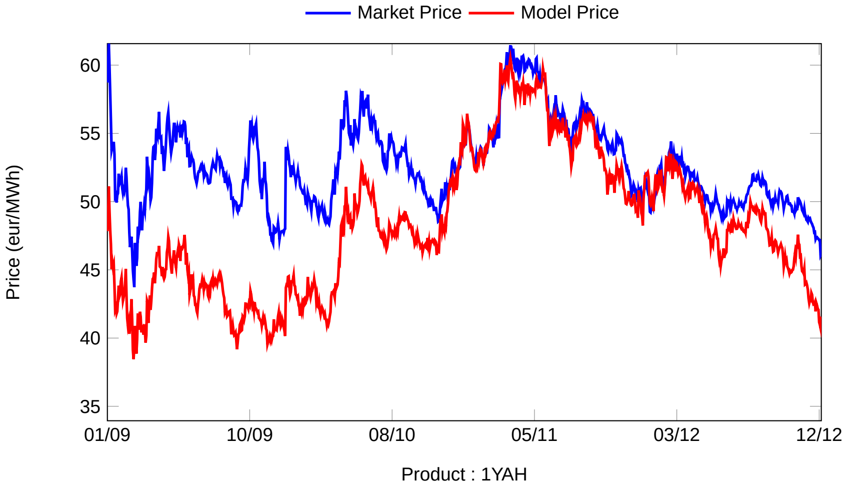


Figure : 1-Year-ahead reconstruction: observed prices (blue) and reconstructed forward prices from complete structural model with (red) and without (pink) the scarcity function

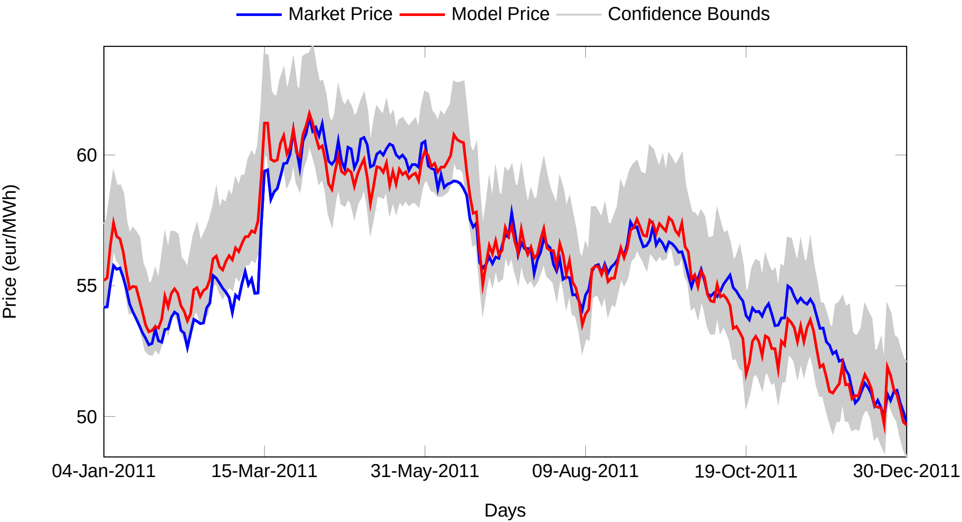
Forward reconstruction with model uncertainty

$$f_D(t) = d_1^{(D)} + d_2^{(D)} \cos\left(2\pi(t - d_3^{(D)})\right) + \mathbf{week}_D(t)$$
$$f_i(t) = d_1^{(i)} + d_2^{(i)} \cos\left(2\pi(t - d_3^{(i)})\right) + \mathbf{week}_i(t)$$

$$dZ_D(t) = -\alpha_D Z_D(t)dt + \beta_D dW_t^D$$
$$dZ_i(t) = -\alpha_i Z_i(t)dt + \beta_i dW_t^i$$

- Parameters of demand and capacities models are estimated on historical data
- Confidence intervals can also be estimated
- A confidence interval can be dedeuced on forward prices
  - from 20 model parameters
  - with 400 Monte Carlo simulation on each parameter's confidence interval

Forward reconstruction with model uncertainty



Calibration

- Calibration problem
  - At a given date t, we observe several forward products $(F_t(T,\theta))_(T,\theta)$
  - Objective of calibration: determine model parameters to exactly retrieve all the forward products
- Impossible for the current model
- Introduction of a "risk premium" $\varepsilon(T)$ in the demand process

$$
\boxed{\begin{aligned}
f_D(t) &= \varepsilon(T) + d_1^{(D)} + d_2^{(D)} \cos\left(2\pi(t - d_3^{(D)})\right) + \mathbf{week}_D(t) \\
f_i(t) &= d_1^{(i)} + d_2^{(i)} \cos\left(2\pi(t - d_3^{(i)})\right) + \mathbf{week}_i(t)
\end{aligned}}
$$

Calibration results

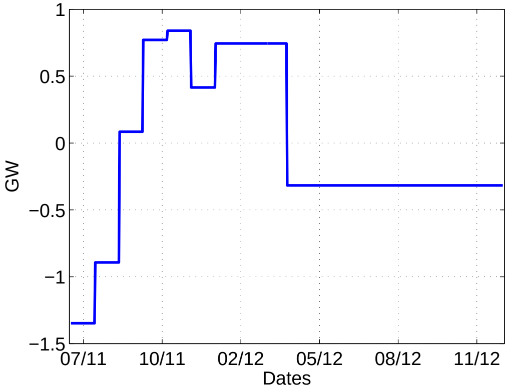


Figure : Calibrated $\varepsilon(T)$ on observed forward prices: 1MAH$\rightarrow$ 6MAH, 1QAH$\rightarrow$ 3QAH and 1YAH observed on 28-Jun-2011.

Calibration results

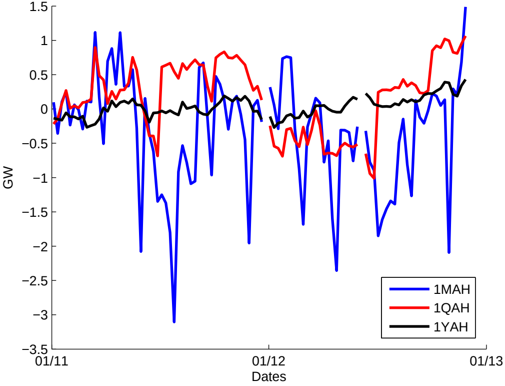


Figure : Evolution of $\varepsilon(T)$ over time

Conclusion

- **Satisfactory reconstruction of forward products**
  - Enough random factors to produce non trivial long term prices
  - Long term products seem to be strongly linked with fundamentals
  - A risk aversion is observable
- **The calibration on observed forward products is possible**
  - The "implied" function $\varepsilon(T)$ is reasonable
  - Short-term forward products present more risk aversion
- **Perspectives**
  - **2-factor model**
    - Keep volatility functions $\sigma_S(t)$ and $\sigma_l(t)$
    - Semi-parametric estimation
    - Some first statistical results
    - improve the link between commodities
  - **Structural model**
    - Deepen the parameter uncertainty propagation
    - Calibration on options

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Results: Earnings at Risk (UK Power)

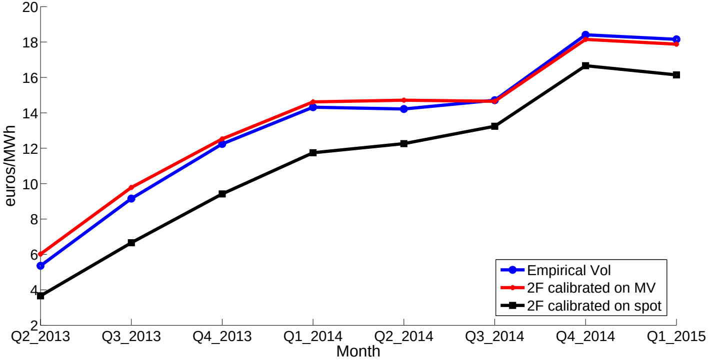

Figure : Earnig-at-Risk estimation

Results: Estimation on Fench Power

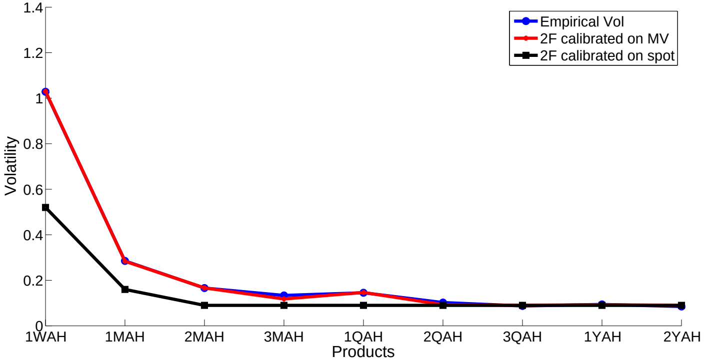

Figure : Estimation results on French power: volatility reconstruction

parameter	Estimation on MV	Estimation on spot
σ_S	45%	302%
α	8.73	88.15
σ_l	11%	11%

Results: European options pricing (French Power)

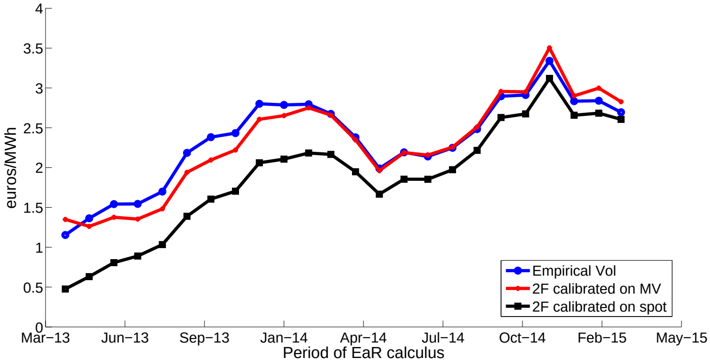

Figure : European option pricing on French Power

	Empirical volatility	Estimation on MV	Estimation on spot
MtM (€)	41 068	40 326	27 649
Relative difference	–	-2%	-33%

Results: Earnings at Risk (French Power)

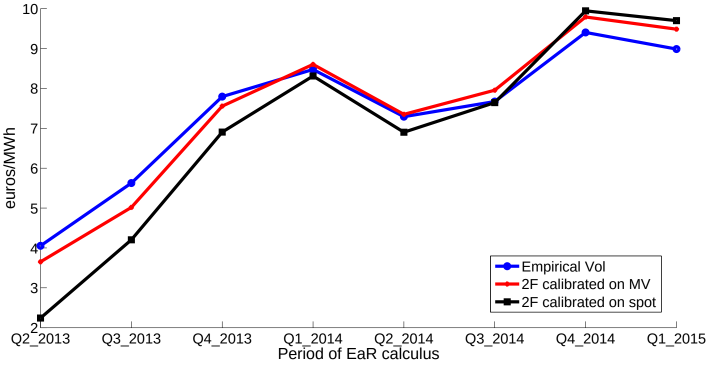

Figure : Earning-at-Risk estimation

Recent work on parameters estimation

- Model formulation: keep $\sigma_s(t)$ and $\sigma_l(t)$ as function.

$$\frac{dF_t(T)}{F_t(T)} = \sigma_s(t)e^{-\alpha(T-t)}dW_t^{(s)} + \sigma_l(t)dW_t^{(l)}$$

- Approximation $F_t(T, \theta) = F_t(T)$ $\Rightarrow$ statistical (asymptotic) results on estimates

- Semi-parametric estimation
  - $\sigma_s(t)$ and $\sigma_l(t)$ are not parametrized
  - Focus on estimation of α (considering $\sigma_s(t)$ and $\sigma_l(t)$ as nuisance parameters)
  - Non parametric estimation of functions $\sigma_s(t)$ and $\sigma_l(t)$

Identification and estimation with two maturities

- Observations $(F_{t_n}(T_1))_{t_n \in [0\ ;\ T_1]}$ and $(F_{t_n}(T_2))_{t_n \in [0\ ;\ T_2]}$ of two forward products of respective delivery T_1 and T_2.
  We will suppose $T_1 < T_2$ and $t_N = T_1$ (i.e. the number of jointly observale forward products is N).
- Let us note $\Delta F_i^n(T) = \ln\left(\frac{F_{t_i}(T)}{F_{t_{i-1}}(T)}\right)$
- Estimation of α
  - Define the function $g : (-1,1) \ni x \mapsto -\frac{1}{T_2-T_1} \ln\left(\frac{1+x}{1-x}\right) \in \mathbf{R}$.
  - Let $\hat{q} = \frac{\sum_{i=1}^n \left(\Delta F_i^n(T_2) - \Delta F_i^n(T_1)\right)^2}{\sum_{i=1}^n \left(\left(\Delta F_i^n(T_2)\right)^2 - \left(\Delta F_i^n(T_1)\right)^2\right)}$.
  - Estimator of α $\boxed{\hat{\alpha} = g\left(\hat{q}\mathbf{1}_{\hat{q}\in(-1,1)}\right)}$

Identification and estimation with two maturities

Theorem

Suppose

- σ_s and σ_l are continuous on $[0\ ;\ T_1]$,
- $\exists\,(\kappa, K) \in (\mathbf{R}_*^+)^2, \forall i = 1, ..., n, \kappa \frac{T_1}{n} \leq |t_i - t_{i-1}| \leq K \frac{T_1}{n}$

Then $\sqrt{n}\,(\hat{\alpha} - \alpha) \xrightarrow[n\to+\infty]{} \mathcal{N}(0, V_\infty(\alpha))$ with

$$V_\infty(\alpha) = \frac{T_1}{(T_2 - T_1)^2}\left(e^{\alpha(T_2 - T_1)} - 1\right)^2 \frac{\int_0^{T_1} e^{-2\alpha(T_1 - t)}\sigma_s^2(t)\sigma_l^2(t)\mathrm{d}H_t}{\left(\int_0^{T_1} e^{-2\alpha(T_1 - t)}\sigma_s^2(t)\mathrm{d}t\right)^2} \qquad (1)$$

with $H_t = \lim_{n\to+\infty} \frac{n}{T_1} \sum_{t_i \leq t}(t_i - t_{i-1})^2$ the Asymptotic Quadratic Variation of Time.

- By noting $T_2 = T_1 + \tau$, we have $V_\infty(\alpha) \xrightarrow[T_1\to\infty]{} \frac{2\alpha(e^{\alpha\tau} - 1)^2}{\tau^2} \frac{\sigma_l^2}{\sigma_s^2} T_1$
  - When $T_1 \to \infty$, the behavior of $V_\infty(\alpha)$ is linear in T_1
  - $V_\infty(\alpha)$ increases when α increases