UNIVERSITÄT DUISBURG ESSEN

A Structural Model for Interconnected Electricity Markets

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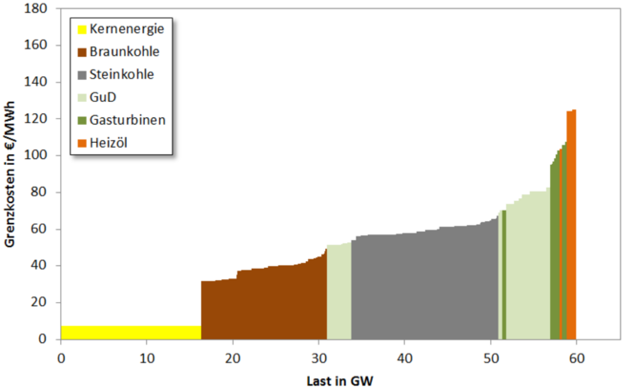
The Merit Order

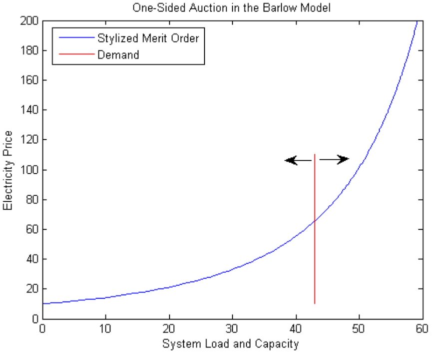


From: Forschungsstelle fuer Energiewirtschaft e. V. (FfE)
Not continuous, piecewise 'constant', differences in production technology.

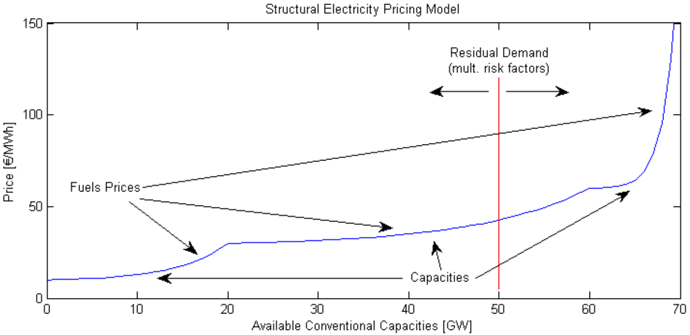
Simple Example of a Structural Model

Barlow (2002) uses a simple parametrization of the merit order:

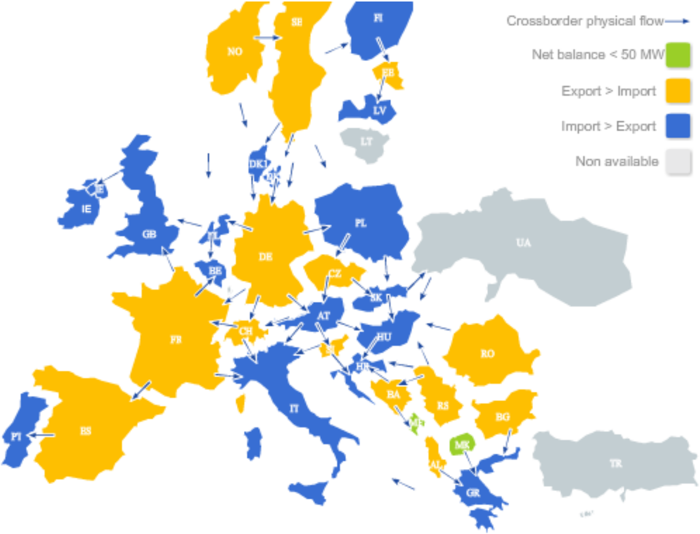
$$dD_t = \kappa(\theta - D_t)dt + \sigma dW_t$$
$$C_t(P_t) = a_0 - b_0(1 + \alpha P_t)^{\beta}$$



Typical Structural Model - One Market



The European Electricity Market



Interconnector Capacity - Germany

Net transfer capacity (NTC) as reported by ENTSO-E during Winter 2010/11 for peak-hours (in MW).

Country	FR	NL	DK	SE	CH	AT	Total
Import from	2700	3000	2000	600	3500	2000	13800
Export to	3200	3800	1500	600	1500	2200	12800

Price Variablility due to Interconnectors

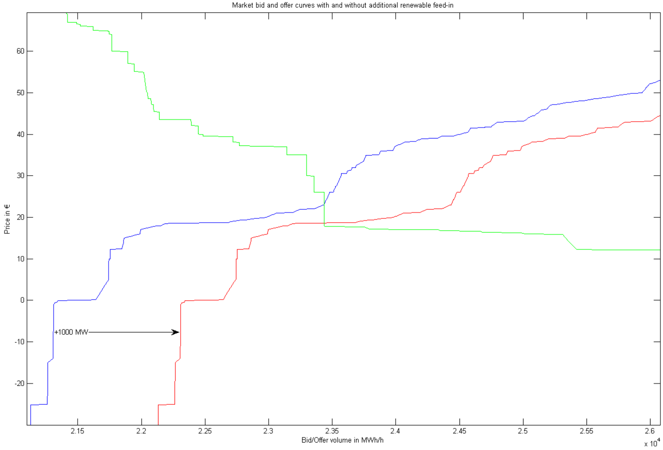


Figure: Shift in Offer due to Interconnectors

Economic Assumptions

1. Marginal demand is normal but correlated over countries (as in Aid, Coulon, Barlow, ...)
2. Heat rates are exponential in capacity used (as in Coulon, Elliot, ...)
3. Market supply curve is piecewise (per fuel) affine-linear in fuels prices (generalization of Aid, Coulon, ...)

Mathematical Assumptions

We assume a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ which supports our model.
On this space, we have:
$W_t^D = (W_t^1, \dots, W_t^n)$ the brownian motion which is driving demand (generating $\mathcal{F}^D = (\mathcal{F}_t^D)_t$).
$W_t^S = (W_t^{S_1}, \dots, W_t^{S_m})$ the brownian motion driving fuels prices (generating $\mathcal{F}^S = (\mathcal{F}_t^S)_t$).
$\mathcal{F}_t = \mathcal{F}_t^D \vee \mathcal{F}_t^S$, $\mathbb{F} = (\mathcal{F}_t)_t$ the market filtration which consists of all information contained in fuels prices and demand.

The Model

For the sake of simplicity, we assume that only one fuel is marginal per electricity market.
We have country $i, j \in \{1, \ldots, n\}$ and a (common) market minimum price of $c \in \mathbb{R}$.
For each country i, we assume a market supply curve of the following form:

$$C^i(x, s) = s e^{a_i + b_i x} + c$$

Demand is given by

$$D_t^i = f_t^i + \tilde{D}_t^i, \quad d\tilde{D}_t^i = k^i(\theta^i - \tilde{D}_t^i)dt + \sigma^i dW_t^i, \quad dW_t^i dW_t^j = \rho_{ij} dt$$

where f_t^i denotes the seasonal component.
We have

1. D is independent from $\mathcal{F}^S$
2. $D_t|_{\mathcal{F}_s^D} \sim N(\mu(s,t), \Sigma(s,t))$ and we will omit the arguments of μ and Σ in the sequel.

Cross Border physical Flows

We denote the physical flow from country j to country i by

$$E_t^{ij}, \ \forall 1 \leq i < j \leq n.$$

The maximum capacity is restricted and might depend on the direction of the flow:

$$E_t^{ij} \in \left[E_{min}^{ij}, E_{max}^{ij}\right], \ E_{min}^{ij} < 0, \ E_{max}^{ij} > 0.$$

Cross Border physical Flows - two Market case

Note that in the two market case (i.e. only E_t^{12} exists), if

- $E_{min} = E_{max} = 0$, markets are not connected and thus, pricing might be done independently.
- $E_{max} = -E_{min} \to \infty$, the interconnector is never congested and thus, one unique market price for both markets exists at all hours.

For the rest of the talk, we will consider the two market case only.

Cross Border physical Flows - two Market case II

In interconnected markets, only the electricity which is not imported has to be produced. Thus, the electricity price is determined as

$$P_t^i(D_t^i, E_t, S_t) = C^i(D_t^i - E_t, S_t).$$

Here, E_t is the imported amount and $D_t^i - E_t$ is the residual demand which has to be satisfied by local production.
Define:

$$\begin{aligned}
A_1 &= \{\omega \in \Omega : P_t^1(D_t^1, E_{max}, S_t) \geq P_t^2(D_t^2, -E_{max}, S_t)\} \\
A_2 &= \{\omega \in \Omega : P_t^1(D_t^1, E_{min}, S_t) \leq P_t^2(D_t^2, -E_{min}, S_t)\} \\
A_3 &= \Omega \setminus (A_1 \cup A_2)
\end{aligned}$$

Then, the cross border flow is

$$E_t^{12}(\omega) = \begin{cases} E_{max} & \text{, if } \omega \in A_1 \\ E_{min} & \text{, if } \omega \in A_2 \\ \frac{a_1 - a_2}{b_1 + b_2} + \frac{b_1}{b_1 + b_2} D_t^1(\omega) - \frac{b_2}{b_1 + b_2} D_t^2(\omega) & \text{, if } \omega \in A_3 \end{cases}$$

Market Clearing Prices

Given the cross border physical flow which minimizes price differences between countries, the resulting electricity price for country 1 may be stated as follows:

$$P_t^1(\omega) = P_t^1(D_t^1, E_t^{12}, S_t) = \begin{cases} C^1(D_t^1(\omega) - E_{max}, S_t(\omega)) & \text{, if } \omega \in A_1 \\ C^1(D_t^1(\omega) - E_{min}, S_t(\omega)) & \text{, if } \omega \in A_2 \\ C^m(D_t^1(\omega) + D_t^2(\omega), S_t(\omega)) & \text{, if } \omega \in A_3 \end{cases}$$

with C^m as specified on the next slide. Equivalent results hold for P_t^2 in country 2.

Supply curve in the case of market convergence

In the case of market convergence, aggregated demand has to be met by the cheapest production units in both countries. For S_t fixed, we thus define

$$(C^i)^{-1}(y, S_t) = \inf\{x \in \mathbb{R} : C^i(x, S_t) \geq y\}$$

and find the aggregated supply curve as

$$C^m(x, S_t) = \inf\{y \in \mathbb{R} : (C^1)^{-1}(y, S_t) + (C^2)^{-1}(y, S_t) \geq x\}$$

which has the form

$$C^m(x, s) = se^{a_m + b_m x} + c$$

with $a_m = \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2}$ and $b_m = \frac{b_1 b_2}{b_1 + b_2}$.

Distribution of the market clearing price - limiting cases

Assuming lognormal fuels prices, i.e. $\log(S_t)|_{\mathcal{F}_s^S} \sim N(\mu^S, \sigma_S^2)$, we define the generalized lognormal distribution $logN(\mu, \sigma^2, c)$ as the distribution of a absolutely continous random variable X with density

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma(x-c)} e^{-\frac{1}{2}\left(\frac{\ln(x-c)-\mu}{\sigma}\right)^2}, \forall x \in (c, \infty)$$

then, it obviously holds that

$$P_t^i|_{\mathcal{F}_s} \xrightarrow{d} logN(\mu^S + a_i + b_i\mu_i, \sigma_S^2 + b_i^2\sigma_i^2, c) \text{ as } E_{max} = -E_{min} \to 0^+$$

$$P_t^i|_{\mathcal{F}_s} \xrightarrow{d} logN(\mu^S + a_m + b_m(\mu_1 + \mu_2), \sigma_S^2 + b_i^2(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2), c)$$
$$\text{as } E_{max} = -E_{min} \to \infty$$

Distribution of the market clearing price

In the case of known fuels prices (realistic over short time horizon), we calculate:

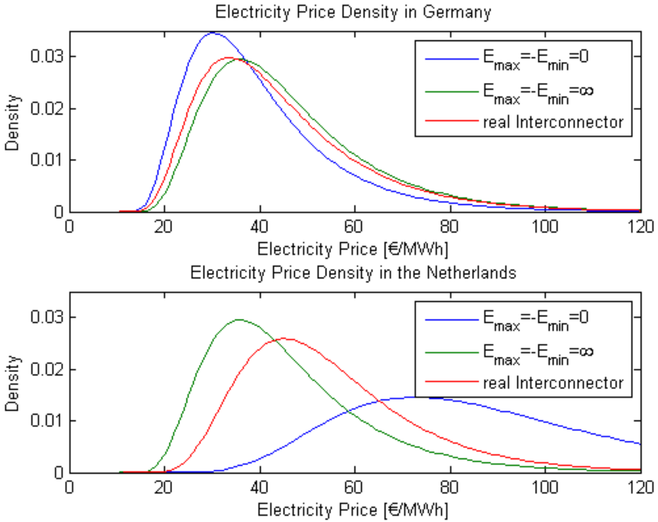
$$F_{P_t^1|\mathcal{F}_s}(x) = \mathbb{Q}(P_t^1 \leq x|\mathcal{F}_s) =$$
$$\mathbb{Q}(\{P_t^1 \leq x\} \cap A_1|\mathcal{F}_s) + \mathbb{Q}(\{P_t^1 \leq x\} \cap A_2|\mathcal{F}_s) + \mathbb{Q}(\{P_t^1 \leq x\} \cap A_3|\mathcal{F}_s).$$

We are able to calculate above Probabilities. It turns out

$$\mathbb{Q}(\{P_t^1 \leq x\} \cap A_1|\mathcal{F}_s) =$$
$$\Phi_2\left(\begin{bmatrix} \frac{\ln(x-c)-a_1}{b_1} - \mu_1 + E_{max} \\ a_1 - a_2 - (b_1+b_2)E_{max} + b_2\mu_2 - b_1\mu_1 \end{bmatrix}; \begin{bmatrix} \sigma_1^2 & b_2\rho\sigma_1\sigma_2 - b_1\sigma_1^2 \\ b_2\rho\sigma_1\sigma_2 - b_1\sigma_1^2 & b_2^2\sigma_2^2 - 2b_1b_2\rho\sigma_1\sigma_2 + b_1^2\sigma_1^2 \end{bmatrix}\right)$$

and similar (i.e. the differences of bivariate normal expressions with constant covariance matrix evaluated at affine-linear transformations of a vector which depends on the shifted logarithm of x) for the other 2 terms.

Distribution of the market clearing prices in GER and NL



Futures prices in the structural model

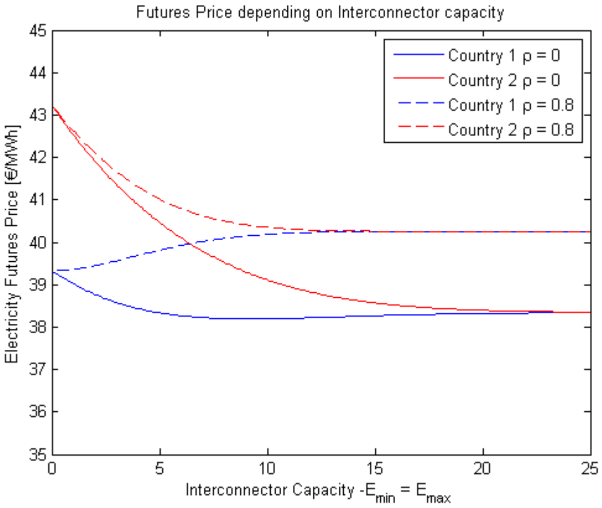
We first consider futures with immediate delivery. Denote by $F^i(s,t)$ the futures price of electricity in country i at time s for delivery in t. Under the risk-neutral measure we should have

$$F^1(s,t) = \mathbb{E}_s^Q[P_t^1] = \int_\Omega P_t(\omega)\mathbb{Q}(d\omega)$$
$$= \int_{A_1} P_t(\omega)\mathbb{Q}(d\omega) + \int_{A_2} P_t(\omega)\mathbb{Q}(d\omega) + \int_{A_3} P_t(\omega)\mathbb{Q}(d\omega)$$

and again, assuming deterministic fuels prices, we find

$$\int_{A_1} P_t(\omega)\mathbb{Q}(d\omega) = c\Phi\left(\frac{a_1 - a_2 - E_{max}(b_1+b_2) + b_1\mu_1 - b_2\mu_2}{\sqrt{b_1^2\sigma_1^2 - 2b_1b_2\rho\sigma_1\sigma_2 + b_2^2\sigma_2^2}}\right) +$$
$$e^{a_1+b_1(\mu_1-E_{max})+\frac{b_1^2\sigma_1^2}{2}}\Phi\left(\frac{a_1 - a_2 - E_{max}(b_1+b_2) + b_1\mu_1 - b_2\mu_2 + b_1^2\sigma_1^2 - b_1b_2\rho\sigma_1\sigma_2}{\sqrt{b_1^2\sigma_1^2 - 2b_1b_2\rho\sigma_1\sigma_2 + b_2^2\sigma_2^2}}\right)$$

Futures prices depending on Interconnector capacity



Thank you for your attention...

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Appendix

Price Convergence FR - GER (hourly basis)

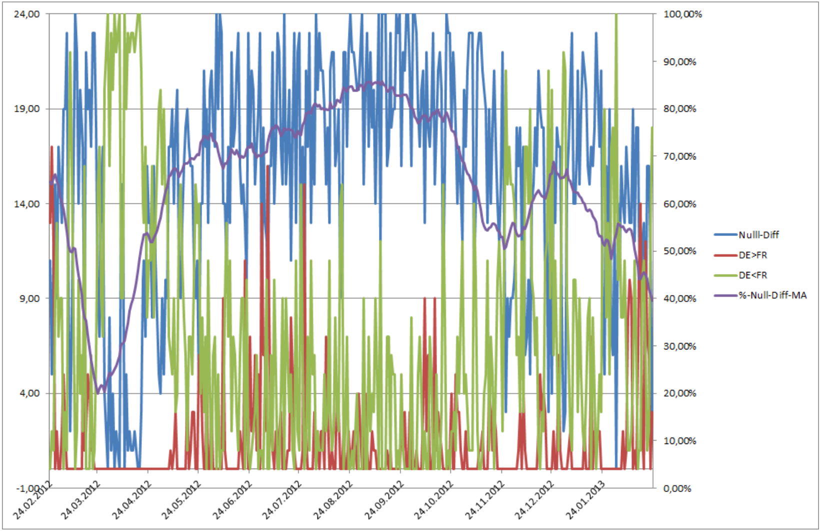

Figure: Price Convergence between France and Germany