The Correlation Structure of Commodity Prices and Exchange Rates

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Workshop on Stochastic Games, Equilibrium, and Applications to Energy & Commodity Markets
Fields Institute, August 2013

Background for paper

- Whys is this correlation an issue?
  - Participants in commodity markets will be exposed to both commodity price risk and FX risk. The valuation of portfolios, exposures etc. will depend on the relationship between the commodity price and the FX rate.
- Are FX rates and commodity prices interlinked
  - Several studies show a link between commodity prices and FX rates
- How are the linked?
  - Financial news often put "rising commodity prices" and "weakened dollar" in the same sentence and research mainly focuses on causality to which extent changes in one explains changes in the other.

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- The general perception is that the correlation is positive, i.e., there is less uncertainty in the value of the position measured in EUR.
- Even in a simple BS-world: The value of oil measured in EUR depends on the constant parameters, including the correlation.
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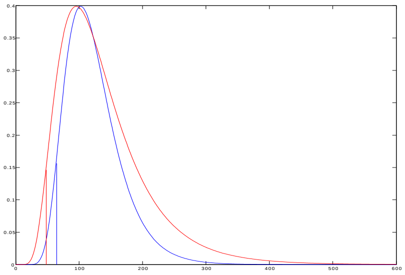
Correlation matters even in the most simple case:

- Consider a EUR-denominated investor in commodities. He is exposed to both commodity price changes (measured in USD) and to changes in the EURUSD rate (the cost of one EUR measured in USD)
- He holds one unit of the commodity and he wants to calculate his 1-year Value-at-Risk of the position
- He sets up a simple Black-Scholes model: E_t denotes the commodity price and X_t is the spot FX rate. The returns are correlated by $\mathrm{corr}(W_t^E, W_t^X) = \rho dt$ and the $\mathbb{Q}^D$-dynamics are:

$$\begin{aligned} dE_t &= E_t \left[(r - c)\, dt + \sigma_E dW_t^E \right] \\ dX_t &= X_t \left[(r - r^*)\, dt + \sigma_X dW_t^X \right] \end{aligned}$$

A simple illustration of VaR



Distribution of value of E_1/X_1 with

$\rho < 0$ (*red line*) $\rho > 0$ (*blue line*)

This implies

- positive correlation $\Rightarrow$ narrow distribution, i.e., there is a certain degree of self-hedge
- negative correlation $\Rightarrow$ wide distribution, i.e., more potential upside AND downside
- Thus; correlation matters!

Content of paper

- Investigate the observed oil-FX correlation structure
- Introduce and estimate a model for the term structure of commodity prices and FX rates, that
  - allows for stochastic correlation
  - captures stochastic volatility
  - match futures/forward price curves
  - match option prices
- Currency basket that minimizes oil price volatility
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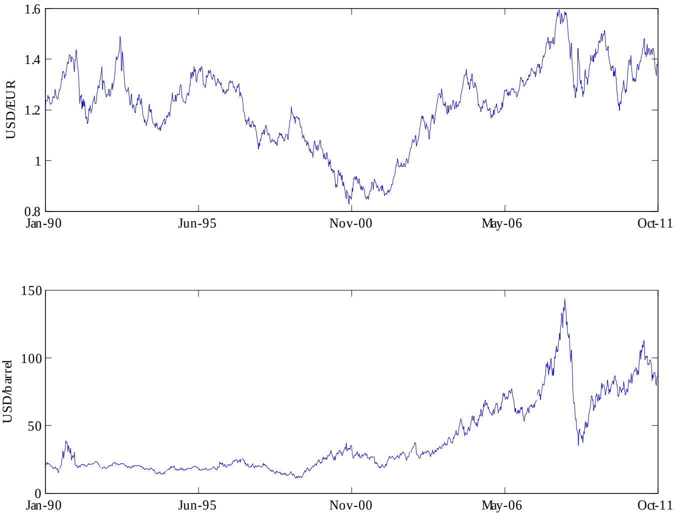
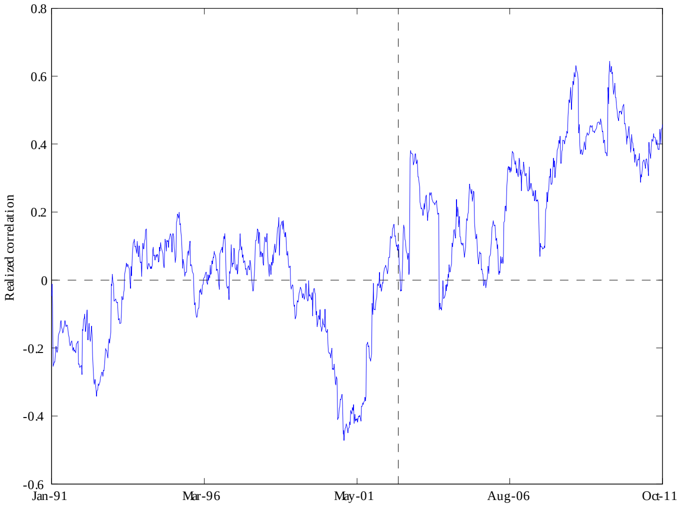
Literature

- Trolle, A. & Schwartz, E. (Review of Financial Studies, 2009): "Unspanned stochastic volatility and the pricing of commodity derivatives"
- Pilz, K. F. & Schlögl E.: "A Hybrid Commodity and Interest Rate Market Model"
- Hansen, T. L. & Jensen, B. A. (Copenhagen Business School, working paper, 2004-10): "Energy Options in an HJM Framework"
- Dimitroff, G., Szimayer, A. & Wagner, A. (Fraunhofer ITWM, working paper, 2009): "Quanto Option Pricing in the Parsimonious Heston Model"
- Several economic/econometrics papers on the link between FX and oil

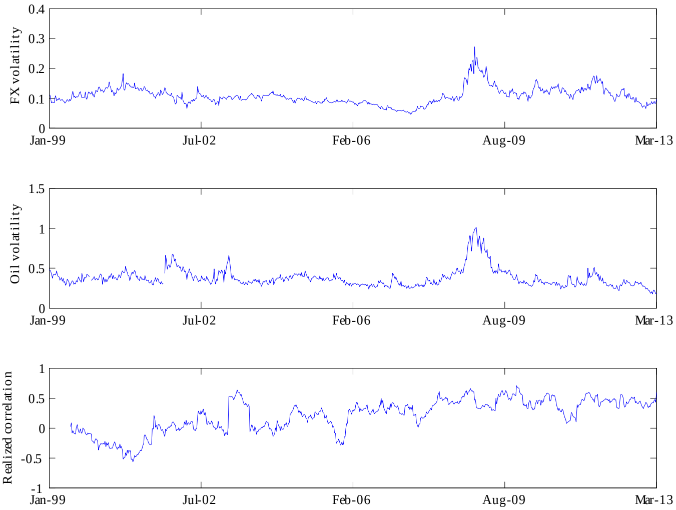
Spot FX and closest (liquid) WTI futures contract



13-week rolling correlation (returns)



Realized correlation and volatility relates



Cointegration and motivation for model

- Is there a cointegrating relation between the prices?
  - No. Reduced rank is rejected.
- Are there one or more cointegrating relations among the correlation and volatility processes?
  - Yes. Reduced rank is not rejected, indicating two driving factors for the correlation and volatility processes.

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Model

Commodity price process

$$\frac{dE_t}{E_t} = \delta_t^E dt + \sigma_{E1}\sqrt{\upsilon_t^1}dW_t^{E,1} + \sigma_{E2}\sqrt{\upsilon_t^2}dW_t^{E,2} \quad (1)$$

$$dy_t^E(T) = \mu_y^E(t,T)dt + \sigma_y^E(t,T)\sqrt{\upsilon_t^1}dW_t^{E,3} \quad (2)$$

FX rate

$$\frac{dX_t}{X_t} = \delta_t^X dt + \sigma_{X1}\sqrt{\upsilon_t^1}dW_t^{X,1} + \sigma_{X2}\sqrt{\upsilon_t^2}dW_t^{X,2} \quad (3)$$

$$dy_t^X(T) = \mu_y^X(t,T)dt + \sigma_y^X(t,T)\sqrt{\upsilon_t^2}dW_t^{X,3} \quad (4)$$

Volatility terms for $(i,j) = (1,2)$ and $(2,1)$

$$d\upsilon_t^i = \left(\eta_i - \kappa_i\upsilon_t^i - \kappa_{ij}\upsilon_t^j\right)dt + \sigma_{\upsilon^i}\sqrt{\upsilon_t^i}dZ_t^i \quad (5)$$

Model

- The most important correlations are between the driving Brownian motions of the spot commodity and spot FX rate:

$$corr\left(dW_t^{E,i}, dW_t^{X,i}\right) = \rho_i^{EX} dt \quad with\ \rho_1^{EX}\rho_2^{EX} < 0$$

- I.e., the model yields the following instantaneous covariance of the returns:

$$cov_t\left(\frac{dE_t}{E_t}, \frac{dX_t}{X_t}\right) = \left(\underbrace{\sigma_{E1}\sigma_{X1}\upsilon_t^1\rho_1^{EX}}_{>0} + \underbrace{\sigma_{E2}\sigma_{X2}\upsilon_t^2\rho_2^{EX}}_{<0}\right) dt$$

Futures prices are affine

When $\sigma_y^i(t,T) = \alpha_i e^{-\gamma_i (T-t)}$, the futures price is affine in various state variables:

$$
\begin{aligned}
\log F_t^i(T) &= \log F_0^i(T) - \log F_0^i(t) + \log i_t \\
&\quad + A_i(T-t)\varphi_t^i + B_i(T-t)\phi_t^i \\
d(\log i_t) &= \left[y_0^i(t) + \varphi_t^i + \phi_t^i - \frac{1}{2}\sigma_{i1}\upsilon_t^1 - \frac{1}{2}\sigma_{i2}\upsilon_t^2 \right] dt \\
&\quad + \sigma_{i1}\sqrt{\upsilon_t^1} dW_t^{i,1} + \sigma_{i2}\sqrt{\upsilon_t^2} dW_t^{i,2} \\
d\varphi_t^E &= \left(\left(\rho_{13}^E \sigma_{E1} + \frac{\alpha_E^2}{\gamma_E} \right) \upsilon_t^1 - \gamma_E \varphi_t^E \right) dt + \alpha_E \sqrt{\upsilon_t^1} dW^{E,3} \\
d\phi_t^E &= \left(\frac{\alpha_E^2}{\gamma_E} \upsilon_t^1 - 2\gamma_E \phi_t^E \right) dt
\end{aligned}
$$

Option prices

$$\begin{aligned}\mathcal{P} &= E_t^Q\left(e^{-\int_t^{T_0} r_s ds}(K - F_{T_0}^i(T_1))^+\right)\\ &= P(t, T_0)\left(KG_{0,1}(\log K) - G_{1,1}(\log K)\right)\\ &\quad +cov^Q\left(e^{-\int_t^{T_0} r_s ds}, (K - F_{T_0}^i(T_1))^+\right)\end{aligned}$$

where

$$G_{a,b}(y) = \frac{\psi(a, t, T_0, T_1)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left(\psi(a + \mathrm{i}ub, t, T_0, T_1)\, e^{-\mathrm{i}uy}\right)}{u} du$$

and $\psi(u, t, T_0, T_1) = E_t^Q\left(e^{u \log F_{T_0}^i(T_1)}\right)$ is determined by a system of ODEs involving the parameters in the model.

Available data

- WTI crude oil
  - Daily closing prices from January 1990 to March 2013
  - Futures contracts spanning all months for the next 5-6 years and semi-annual spanning three years after that
  - Using weekly observations of six monthly contracts, two quarterly and four annual contracts from 1999-2013
- Exchange rate
  - Spot EURUSD from January 1990 to March 2013
  - Futures contracts on EURUSD from 1998 to March 2013
  - Using weekly observations of five futures contracts spanning more than a year
- Options
  - (close to) ATM options with short time to maturity. (Not yet included in the estimation)

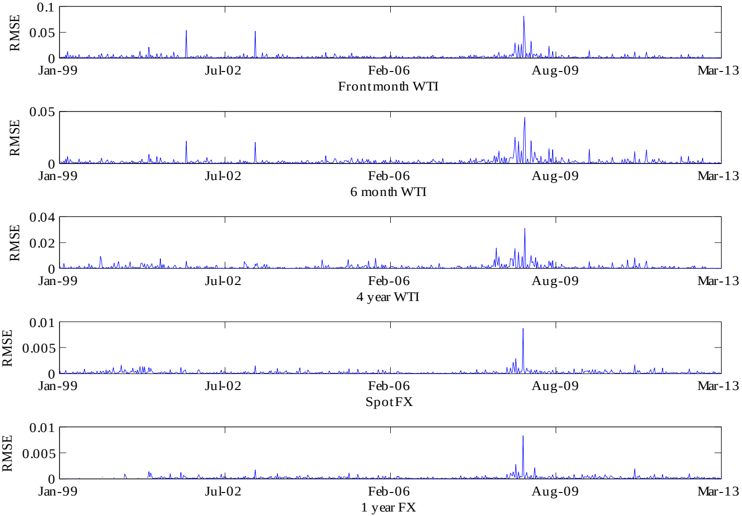
Selected parameter estimates (preliminary)

	Oil	FX
σ_{i1}	0.3682	0.0887
σ_{i2}	0.2026	0.2120
α_i	0.4262	0.0871
γ_i	0.5890	0.0725
κ_i	3.5125	5.9743
κ_{ij}	-2.1160	-0.1463
σ_{υ^i}	8.915	3.535
ρ^i	-0.7747	-0.0788
σ_{fut}	0.0059	0.0002
σ_{spot}	N/A	0.003

$$
\begin{aligned}
\frac{di_t}{i_t} &= \delta_t^i dt + \sigma_{i1}\sqrt{\upsilon_t^1} dW_t^{i,1} \\
&\quad + \sigma_{i2}\sqrt{\upsilon_t^2} dW_t^{i,2} \\
dy_t^i(T) &= \ldots + \alpha_i e^{-\gamma_i(T-t)}\sqrt{\upsilon_t^i} dW_t^{i,3} \\
d\upsilon_t^i &= \left(\eta_i - \kappa_i \upsilon_t^i - \kappa_{ij}\upsilon_t^j\right) dt \\
&\quad + \sigma_{\upsilon^i}\sqrt{\upsilon_t^i} dZ_t^i
\end{aligned}
$$

ρ_1^{EX} : **0.5478**, ρ_2^{EX} : **−0.4686**

Selected RMSE plots



Recap

- Even in a Black-Scholes world correlation matters
- Correlation is not directly observable, but using realized correlation based on a rolling window, data shows a time changing correlation of both signs
- The model captures stochastic correlation with changing signs as seen in the markets.
- With the model parameters, the simulated correlation picture seems suitable compared to historic correlation
- The correlation surface plots shows numerically decreasing correlation in time to maturity similar to what is seen in data.

Thank you