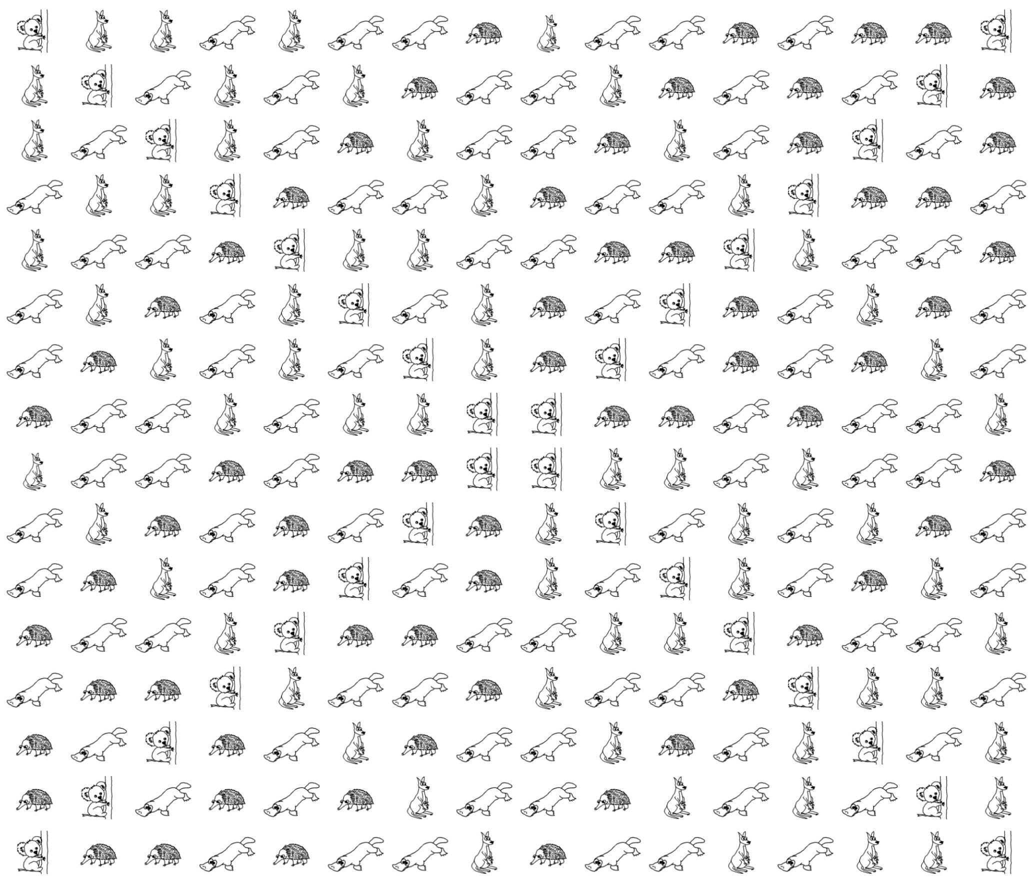
Algebraic Combinatorics:
Spectral Graph Theory, Erdös-Ko-Rado Theorems and Quantum Information Theory



A conference to celebrate the work of Chris Godsil
June 23-27 2014

Department of Combinatorics & Optimization
University of Waterloo

CONFERENCE PROGRAM

Monday, June 23

8:00 – 8:45	Registration and coffee
8:45 – 9:00	Opening address
9:00 – 9:55	*Counting edge-coloured regular graphs* Brendan McKay
10:00 – 10:30	Coffee
10:30 – 10:55	*Unique vector colorings: core concepts* David Roberson
11:00 – 12:00	*Chromatic roots of graphs and matroids* Gordon Royle
12:00 – 2:00	Lunch
2:00 – 2:25	*Quantum walk, state transfer, and complex graphs* Christino Tamon
2:30 – 2:55	*On quantum algorithms for difference sets over abelian groups* Martin Roetteler
3:00 – 3:25	*Universal computation by multi-particle quantum walk* Andrew Childs
3:30 – 4:00	Coffee
4:00 – 4:55	*From sandwiches to club sandwiches via quantum ideas* Simone Severini
5:00 – 5:25	*A practical heuristic for finding graph minors* Aidan Roy
5:30 – 7:00	Reception

Tuesday, June 24

8:30 – 9:00	Coffee
9:00 – 9:55	*On tight relative t-designs in association schemes* Eiichi Bannai
10:00 – 10:30	Coffee
10:30 – 10:55	*Permutation codes and their semidefinite programming bound* Peter Dukes
11:00 – 11:55	*A structure theory for graphs with fixed smallest eigenvalue* Jack Koolen
12:00 – 2:00	Lunch
2:00 – 2:25	*Almost distance-regular graphs and walk-regularity* Edwin van Dam
2:30 – 2:55	*On the structure of strongly regular and distance-regular graphs* Sebastian Cioabă
3:00 – 3:25	*Small product sets and edge cuts* Matt DeVos
3:30 – 4:00	Coffee
4:00 – 4:55	*Characterizing friendship* Willem Haemers

Wednesday, June 25

8:30 – 9:00	Coffee
9:00 – 9:55	*Symmetry versus regularity* László Babai
10:00 – 10:30	Coffee
10:30 – 10:55	*The Smith and critical groups of a graph* Qing Xiang
11:00 – 11:55	*Median eigenvalues of graphs* Bojan Mohar
12:15 – 6:00	Excursion to Elora

Thursday, June 26

8:30 – 9:00	Coffee
9:00 – 9:55	*Endomorphisms and synchronization* Peter Cameron
10:00 – 10:30	Coffee
10:30 – 10:55	*Extreme graph symmetries* Marston Conder
11:00 – 11:55	*Symmetry breaking in graphs and groups* Wilfried Imrich
12:00 – 2:00	Lunch
2:00 – 2:25	*New examples of perfect state transfer* Gabriel Coutinho
2:30 – 2:55	*Perfect state transfer on distance regular graphs* Krystal Guo
3:00 – 3:25	*The Manickam-Miklós-Singhi conjectures for sets and vector spaces* Ameera Chowdhury
3:30 – 4:00	Coffee
4:00 – 4:55	*Uniform Mixing in Quantum Walks* Chris Godsil
5:30 – 8:30	Banquet

Friday, June 27

8:30 – 9:00	Coffee
9:00 – 9:55	*Doubly transitive permutation groups: Case closed or Open for business?* Cheryl Praeger
10:00 – 10:30	Coffee
10:30 – 10:55	*Automorphisms of Cayley graphs that respect partitions* Joy Morris
11:00 – 11:55	*Complex Hadamard matrices contained in a Bose-Mesner algebra* Akihiro Munemasa
12:00 – 2:00	Lunch
2:00 – 2:25	*Hamiltonian cycles in some easy Cayley graphs* Dave Morris
2:30 – 2:55	*Computational complexity of maxclique for Cayley graphs* Brendan Rooney
3:00 – 3:25	*Sets of unit vectors with few inner products and distance-regular graphs* Junbo Huang
3:30 – 4:00	Coffee
4:00 – 4:55	*The Erdős-Ko-Rado theorem: an algebraic perspective* Karen Meagher

ABSTRACTS

László Babai

Symmetry versus regularity

Abstract: Symmetry is usually defined in terms of automorphisms; regularity in terms of numerical parameters. Symmetry conditions imply various regularity constraints, but not conversely.

We shall discuss two areas of this relationship.

First we review some old results by Peter Frankl on how symmetry and regularity constraints affect extremal set systems.

Our second topic involves recent results on how regularity constraints restrict the order and the structure of the automorphism group, especially for Steiner designs, strongly regular graphs, and coherent configurations. These results were found by various subsets of Xi Chen (Columbia U), Xiaorui Sun (Columbia U), Shang-Hua Teng (U Southern California), John Wilmes (U Chicago), and the speaker.

Eiichi Bannai

On tight relative t-designs in association schemes

Abstract: The concept of relative t-designs in association schemes was introduced by Delsarte in his paper: Pairs of vectors in the space of an association scheme (1977). This concept, in a sense, predicted the concept of Euclidean t-designs, introduced later by Neumaier and Seidel (1988). However, it seems that the study of Euclidean t-designs has preceded the study of relative t-designs in association schemes, and that the study of the latter has just started recently in a way modeling the study of Euclidean t-designs. In this talk we first review the similarities between the studies on "spherical t-designs and Euclidean t-designs" and on "t-designs and relative t-designs in association schemes", putting the emphasis on the study of tight t-designs in each situation. The purpose of this talk is to try to convince the reader that we should study the classification problems of tight relative t-designs in association schemes in a systematic way. More details will be available in the following papers initiating the study in this direction.

[1] E. Bannai, E. Bannai, S. Suda, and H. Tanaka: On relative t-designs in polynomial association schemes, preprint, arXiv:1303.7163.

[2] E. Bannai, E. Bannai, and H. Bannai: On the existence of tight relative 2-designs on binary Hamming association schemes, Discrete Mathematics 314 (2014), 17–37.

[3] Z. Xiang: A Fisher type inequality for weighted regular t-wise balanced designs, J. Combin. Theory Ser. A 119 (2012), 1523–1527.

[4] Y. Zhu, E. Bannai, and E. Bannai: On tight relative 2-designs on the Johnson association schemes (a tentative title, in preparation).

Peter Cameron

Endomorphisms and synchronization

Abstract: There are two natural correspondences between graph endomorphisms and transformation semigroups. One way round, every graph has a semigroup of endomorphisms, which acts on the set of vertices of the graph. The other way, from a transformation semigroup S we can build a graph $Gr(S)$, having various nice properties: it is complete if and only if S is a permutation group, and null if and only if S is synchronizing; and it has clique number equal to chromatic number. These two correspondences

interact: always $End(Gr(S))$ contains S, and $Gr(End(Gr(M))) = M$.

This point of view has applications in the study of synchronizing automata. In particular, it would be interesting to classify the maximal non-synchronizing semigroups in terms of the corresponding graphs; I have partial results about these, and a conjectured characterization of them.

If time permits, I will also report on some links between synchronization and various topics in extremal combinatorics and finite geometry.

Andrew Childs

Universal computation by multi-particle quantum walk

Abstract: A quantum walk is a time-homogeneous quantum-mechanical process on a graph defined by analogy to classical random walk. The quantum walker is a particle that moves from a given vertex to adjacent vertices in quantum superposition. We consider a generalization of quantum walk to systems with more than one walker. A continuous-time multi-particle quantum walk is generated by a time-independent Hamiltonian with a term corresponding to a single-particle quantum walk for each particle, along with an interaction term. Multi-particle quantum walk includes a broad class of interacting many-body systems such as the Bose-Hubbard model and systems of fermions or distinguishable particles with nearest-neighbor interactions. We show that multi-particle quantum walk is capable of universal quantum computation. Since it is also possible to efficiently simulate a multi-particle quantum walk of the type we consider using a universal quantum computer, this model exactly captures the power of quantum computation. In principle our construction could be used as an architecture for building a scalable quantum computer with no need for time-dependent control.

Based on joint work with David Gosset and Zak Webb.

Ameera Chowdhury

The Manickam-Miklós-Singhi conjectures for sets and vector spaces

Abstract: More than twenty-five years ago, Manickam, Miklós, and Singhi conjectured that for positive integers n, k with $n \geq 4k$, every set of n real numbers with nonnegative sum has at least $\binom{n-1}{k-1}$ k-element subsets whose sum is also nonnegative. We verify this conjecture when $n \geq 8k^2$, which simultaneously improves and simplifies a bound of Alon, Huang, and Sudakov and also a bound of Pokrovskiy when $k < 10^{45}$.

Moreover, our arguments resolve the vector space analogue of this conjecture. Let V be an n-dimensional vector space over a finite field. Assign a real-valued weight to each 1-dimensional subspace in V so that the sum of all weights is zero. Define the weight of a subspace $S \subset V$ to be the sum of the weights of all the 1-dimensional subspaces it contains. We prove that if $n \geq 3k$, then the number of k-dimensional subspaces in V with nonnegative weight is at least the number of k-dimensional subspaces in V that contain a fixed 1-dimensional subspace. This result verifies a conjecture of Manickam and Singhi from 1988.

Joint work with Ghassan Sarkis (Pomona College) and Shahriar Shahriari (Pomona College).

Sebastian Cioabă

On the structure of strongly regular and distance-regular graphs

Abstract: In this talk, I will discuss two problems involving the structure of strongly regular and

distance-regular graphs. The first problem involves the connectivity of the complement of a ball in a distance-regular graph where we generalize the well-known result that the second subconstituent of a primitive strongly regular graph is connected. This is joint work with Jack Koolen. The second problem deals with the extendability of strongly regular graphs; this is the largest integer t with the property that any matching with t edges can be extended to a perfect matching. This is joint work with my Ph.D. student Weiqiang Li.

Marston Conder

Extreme graph symmetries

Abstract: This talk will cover a few examples of graphs having maximum possible symmetry for graphs in their class, including 5-arc-transitive 3-valent graphs, 7-arc-transitive of higher valencies, and locally 9-arc-transitive graphs. One of Chris Godsil's impressive early contributions in this field was his work on GRRs (graphical regular representations). These are essentially Cayley graphs for a group G, having no more automorphisms that are provided by the regular representation of G. Babai and Godsil conjectured in 1982 that almost all finite Cayley graphs have this property. I will show that in contrast, it is possible for Cayley graphs of given valency to have many more automorphisms; indeed for every integer $t > 0$, there are infinitely many Cayley graphs of valency $3^t + 1$ that are 7-arc-transitive.

Gabriel Coutinho

New examples of perfect state transfer

Abstract: A perfect transfer of quantum state in simple graphs is a relatively rare phenomenon. In this talk, we will explore different ways of obtaining examples of graphs admitting perfect state transfer. Our main method consists in combining the adjacency matrix of some graphs into sums of their Kronecker products, thus resulting in a new graph whose quantum spectral properties depend on those of the factors. For instance, we were able to find new examples among direct products of graphs and among switching graphs. This is joint work with Chris Godsil.

Edwin van Dam

Almost distance-regular graphs and walk-regularity

Abstract: A t-walk-regular graph is a graph for which the number of walks of given length between two vertices depends only on the distance between these two vertices, as long as this distance is at most t. Such graphs generalize distance-regular graphs and t-arc-transitive graphs. Moreover, 0-walk-regularity is commonly known as walk-regularity, a concept introduced by Godsil and McKay.

Besides mentioning some generalities about t-walk-regular graphs, we will focus on 1- and in particular 2-walk-regular graphs, and study analogues of certain results that are important for distance-regular graphs. We will for example generalize Delsarte's clique bound to 1-walk-regular graphs and Godsil's multiplicity bound to 2-walk-regular graphs.

Matt DeVos

Small product sets and edge cuts

Abstract: An important problem from additive number theory is that of characterizing sets A, B for which the product set AB is very small. We will highlight a connection between an important special case of this problem and that of finding small edge-cuts in graphs. In particular, we will detail a recent classification of all edge cuts of size at most $2d$ in vertex and edge transitive d-regular graphs, and explain the consequences for product sets. This is joint work with Tom Boothby.

Peter Dukes

Permutation codes and their semidefinite programming bound

Abstract: Observe that the permutations $31\underline{4}2\underline{5}$ and $31\underline{5}2\underline{4}$ differ in two positions, as indicated. Regarded as elements of $\mathtt{Sym}(5)$, their quotient is a transposition.

Now, let D be a collection of conjugacy classes of $\mathtt{Sym}(n)$. A subset $\Gamma \subseteq \mathtt{Sym}(n)$ is an (n, D)-*permutation code* if, for any two distinct elements $\phi, \psi \in \Gamma$, their quotient $\phi\psi^{-1}$ belongs to a class in D. As seen in the above example, this framework permits restriction of the 'Hamming distance' between permutations, which counts disagreeing positions in their single-line presentation.

This talk will essentially divide into two parts. First, I will review the elementary facts and motivation surrounding permutation codes. This includes a brief look at the Bose-Mesner algebra of the conjugacy scheme of $\mathtt{Sym}(n)$ and its linear programming bound. Second, I will introduce the extension to the Terwilliger algebra and give an overview of a recent semidefinite programming attack on upper-bounding permutation codes. It is noteworthy that this attack detects the nonexistence of the projective plane of order six and also helps in two challenging cases with $n = 7$.

This is joint work with Mathieu Bogaerts.

Chris Godsil

Uniform Mixing in Quantum Walks

Abstract: If A is the adjacency matrix of a graph X, we define a *transition matrix* $U(t)$ by

$$U(t) = \exp(itA).$$

For physicists, $U(t)$ determines a continuous quantum walk on the vertices of X. There are a number of interesting questions we can ask about the entries of $U(t)$. In this talk I will be concerned with the following question: For which graphs X is there a time t such that all entries of $U(t)$ have the same absolute value?

If there is such a time, we say that *uniform mixing* occurs on X. The d-cubes provide an infinite family of graphs where uniform mixing does occur, in each case at time $\pi/4$. Currently we know a number of infinite families of graphs that admit uniform mixing, but all such graphs are Cayley graphs for abelian groups of exponent dividing two, three or four. We know that no cycle of even length greater than four and no cycle of prime length greater than three admits uniform mixing. Perhaps the most surprising feature of these results we have is the range of tools used to obtain them: coding theory, association schemes, number theory, algebraic geometry.

My talk will provide an overview of these results, and some of the ideas used in their proofs. This work is joint with Ada Chan, Natalie Mullin, Aidan Roy and Harmony Zhan.

Krystal Guo

Perfect state transfer on distance regular graphs

Abstract: A quantum walk is a quantum process on a graph. We consider a continuous-time quantum walk and a phenomenon called perfect state transfer. We provide necessary and sufficient criteria for distance-regular graphs and, more generally, for graphs in association schemes to have perfect state transfer. Using these conditions, we provide several new examples of perfect state transfer. Joint work with Gabriel Coutinho, Chris Godsil and Frédéric Vanhove.

Willem Haemers

Characterizing friendship

Abstract: We determine all graphs whose adjacency matrix has at most two eigenvalues (multi-plicities included) different from ± 1 and decide which of these graphs are determined bytheir spectrum. This includes the so-called friendship graphs, which consist of a number of edge-disjoint triangles meeting in one vertex. It turns out that the friendship graphis determined by its spectrum, except when the number of triangles equals sixteen.

Joint work with Sebastian M. Cioaba, Jason Vermette and Wiseley Wong.

Junbo Huang

Sets of unit vectors with few inner products and distance-regular graphs

Abstract: A set of unit vectors in $\mathbb{C}^m$ with the property that the standard inner product of distinct vectors in the set has absolute value 0 or α (with $\alpha \neq 0$) is called a $\{0, \alpha\}$-*set*. A vector in $\mathbb{C}^m$ is called *flat* if all of its entries have the same absolute value. In 2005, Godsil and Roy gave a construction of sets of flat vectors using graphs; their construction can be used to construct $\{0, \alpha\}$-sets.

In this talk, I will sketch the construction by Godsil and Roy. I will then present bounds on the sizes of $\{0, \alpha\}$-sets of flat vectors in $\mathbb{R}^m$ and $\mathbb{C}^m$, and I will characterize the distance-regular graphs that can be used to produce $\{0, \alpha\}$-sets that meet these bounds at equality.

Wilfried Imrich

Symmetry breaking in graphs and groups

Abstract: A coloring of the vertices of a graph G is called distinguishing if the trivial automorphism is the only automorphism of G that preserves the coloring. This term was coined by Albertson and Collins 1996. Although the concept was not entirely new, their paper spawned a whole wealth of publications, in particular for finite graphs, but also for infinite ones.

One says such a coloring breaks the symmetries of G. It is not hard to see that the concept can be extended to breaking the action of permutation groups or monoids acting on sets, and to breaking automorphisms and endomorphisms of relational structures.

There is a vast literature on distinguishing finite graphs, but the focus of this talk are symmetries of connected, locally finite, infinite graphs, and connected graphs on countable vertex sets, with occasional excursions to graphs of arbitrary cardinality and symmetries of other structures.

In particular, we will draw attention to a conjecture of Tom Tucker. He conjectured that connected, locally finite, infinite graphs can be distinguished by two colors if every nontrivial automorphism moves infinitely many vertices. This is known as the Infinite Motion Conjecture. This conjecture, its general-

izations to uncountable graphs, to groups acting on structures, and to endomorphisms of countable and uncountable graphs, has become a widely studied topic. Nonetheless, despite many intriguing partial results, it is still open in general.

This talk presents the present status of the conjecture and the methods of proofs used. Moreover, versions on permutation groups with links to topology and metric spaces will be presented, as well as probabilistic aspects.

On the way graphs of arbitrary cardinality will be treated and, for example, results on minimizing one of the color classes in distinguishing colorings with 2 colors.

Jack Koolen

A structure theory for graphs with fixed smallest eigenvalue

Abstract: In 1976, Cameron et al. showed that a connected graph with at least 37 vertices having smallest eigenvalue at least -2 s a generalised line graph. In 1977, Hoffman showed that for all numbers $-2 > \theta > -1-\sqrt{2}$ there exists a number $k((\theta)$ such that any connected graph with smallest eigenvalue at least θ is a generalised line graph provided that its smallest valency is at least $k(\theta)$. In this talk I will present some results for graphs with smallest eigenvalue at least -3. This is joint work with Jaeyoung Yang (POSTECH).

Brendan McKay

Counting edge-coloured regular graphs

Abstract: In the early 1980s, Chris Godsil and the speaker determined an asymptotic formula for the number of Latin rectangles that remains the best. It relies on a theorem of Riordan that relates the matching polynomial of a bipartite graph to the number of perfect matchings in its bipartite complement, which Joni and Rota wrote in the form of an integral. More importantly, it relies on a deep theorem of Godsil that the power sums of the zeros of a matching polynomial count a particular class of closed walks.

Note that a Latin rectangle corresponds to a regular bipartite graph whose edges have been partitioned into perfect matchings. This suggests the question of regular graphs which are not necessarily bipartite, whose edges are similarly partitioned. A integral formula of Godsil for the number of perfect matchings in the complement of an arbitrary graph provided the necessary tool already in 1981, but the necessary non-trivial calculations were not done for almost 30 years, when they were completed by Jeanette McLeod with a little help from the speaker, Paulette Lieby and Ian Wanless.

Karen Meagher

The Erdős-Ko-Rado theorem: an algebraic perspective

Abstract: Several years ago I had the good fortune to have an extremely productive post-doctoral fellowship with Chris. Our work from this period has culminated in a book about the Erdős-Ko-Rado Theorem (that is ϵ way from completion!)

This theorem is a major result in extremal set theory. It gives the exact size and structure of the largest system of sets, with a fixed number of elements, that has the property that any two sets in the system have at least one element in common. There are many extensions of this theorem to combinatorial objects other than set systems, such as vectors subspaces over a finite field, integer

sequences, partitions, and recently, there have been several results that extend the EKR theorem to permutations.

During my post-doc with Chris, we worked on an algebraic approach to proving the EKR theorem for several types of combinatorial objects. This method is the focus of our book and will be the focus of my talk. I will explain this method by showing how it can be used to prove that the natural extension of the EKR Theorem holds for The symmetric group.

Bojan Mohar

Median eigenvalues of graphs

Abstract: The problem of HOMO-LUMO separation that arises in mathematical chemistry is closely related to the magnitude of the median eigenvalues of molecular graphs. This motivates the notion of the median eigenvalue index $R(G)$ which is defined as $max\{|\lambda_h|, |\lambda_l|\}$ where $h = \lfloor (n+1)/2 \rfloor$, $l = \lceil (n+1)/2 \rceil$, and λ_i denotes the ith largest eigenvalue of G. Basic results about the median eigenvalue index eigenvalues of a graph will be presented and their extremal behaviour will be discussed. For example, it can be proved that the median eigenvalues of every connected bipartite graph G of maximum degree at most three belong to the interval $[-1, 1]$ with a single exception of the Heawood graph, whose median eigenvalues are $\pm\sqrt{2}$. Moreover, if G is not isomorphic to the Heawood graph, then a positive fraction of its median eigenvalues lie in the interval $[-1, 1]$. This somewhat surprising result can be extended to higher vertex degrees and provides a motivation for further work. Part of the talk is a joint work with Behruz Tayfeh-Rezaie.

Dave Morris

Hamiltonian cycles in some easy Cayley graphs

Abstract: Some of Chris Godsil's early work (including his Ph.D. thesis) completed the proof of a conjecture of Mark Watkins about automorphism groups of Cayley graphs. We will explain the conjecture, and then we will discuss a different conjecture that is still open: every connected Cayley graph has a hamiltonian cycle. Recent work provides a positive answer when the number of vertices is a small multiple of a prime number, and in some other similar cases.

Joy Morris

Automorphisms of Cayley graphs that respect partitions

Abstract: A Cayley graph $\Gamma = \mathrm{Cay}(G; S)$ on a group G with connection set S, is a graph whose vertices are labelled with the elements of G, with vertices g_1 and g_2 adjacent if $g_1^{-1}g_2 \in S$. We say that an automorphism α of Γ respects the partition $\mathcal{C}$ of the edge set of Γ if for every $C \in \mathcal{C}$, we have $\alpha(C) \in \mathcal{C}$. I will discuss some obvious partitions of the edge set of a Cayley graph Γ, and find conditions under which a graph automorphism of Γ that respects these partitions and fixes a vertex, must be an automorphism of the group G.

Akihiro Munemasa

Complex Hadamard matrices contained in a Bose-Mesner algebra

Abstract: A complex Hadamard matrix is a square matrix H with complex entries of absolute value 1 satisfying $HH^* = nI$, where $*$ stands for the Hermitian transpose and I is the identity matrix of order n. A type-II matrix, or an inverse orthogonal matrix, is a square matrix W with nonzero complex entries satisfying $WW^{(-)^\top} = nI$.

Strongly regular graphs were used to construct type-II matrices by Chan and Godsil. In this talk, we give constructions of type-II matrices and complex Hadamard matrices in the Bose–Mesner algebra of a certain 3-class symmetric association scheme. In particular, we recover the complex Hadamard matrices of order 15 found by Chan. This family was found after extensive computer experiment on the list of 3-class association schemes up to 100 vertices given by Van Dam. Surprisingly, only handful of 3-class association schemes up to 100 vertices, with the exceptions of amorphic or pseudocyclic schemes, admit a complex Hadamard matrix in their Bose–Mesner algebras. We compute the Haagerup sets to show inequivalence of resulting type-II matrices.

This is based on a joint work with Takuya Ikuta.

Cheryl Praeger

Doubly transitive permutation groups: case closed or open for business?

Abstract: A classification of the finite 2-transitive (or doubly transitive) permutation groups became available almost immediately after the finite simple group classification was complete. How usable and useful this classification proved to be, and where it was applied, is quite a story. Part of this story features Chris Godsil, and some of it will be sketched in my lecture.

David Roberson

Unique vector colorings: core concepts

Abstract:

A vector k-coloring of a graph G is an assignment of real unit vectors to the vertices of G such that vectors assigned to adjacent vertices have inner product at most $-1/(k-1)$. The smallest k for which a vector k-coloring exists is known as the vector chromatic number of G. We say that a graph is uniquely vector colorable (UVC) if all of its optimal vector colorings are the same up to orthogonal transformations.

In this talk we will see that all non-bipartite Kneser graphs have unique vector colorings. Interestingly, the vectors used in the coloring are the vertices of the eigenpolytope of the least eigenvalue of the Kneser graph. Our main tool in proving this result is a sufficient condition for uniqueness from the theory of tensegrity frameworks. We will discuss how this condition may be able to be applied to show uniqueness of vector colorings for a much more general class of graphs.

The above mentioned result on tensegrity frameworks can also be used to prove that, under certain conditions, G being UVC and H having vector chromatic number strictly larger than G implies that the categorical product $G \times H$ is UVC.

Lastly, we will discuss how a graph G having a unique vector coloring can sometimes imply that G is a core (has no proper endomorphisms).

This is joint work with Chris Godsil, Brendan Rooney, Robert Šámal, and Antonios Varvitsiotis.

Martin Roetteler

On quantum algorithms for difference sets over abelian groups

Abstract: We address quantum algorithms for the following problem: given quantum oracle access to a function $x \to f(x+s)$, where f is the characteristic function of a difference set in a finite abelian group A and s is an unknown element of A, the task is to determine the hidden shift s while making as few queries as possible to f and to the shifted function.

We discuss a special case of this problem first in which A is the elementary abelian 2-group and f corresponds to a bent function. We show that in this case the shift can be extracted with a constant number of queries, leading to an exponential separation between classical and quantum query complexity. We then explore several possible approaches to this problem to more general Boolean functions f. One of the approaches is based on quantum rejection sampling, a state generation technique whose performance depends on "water-filling" properties of the Fourier spectrum of f. We also discuss possible generalizations to difference sets over other abelian groups besides the Boolean case.

Based on joint work with Andrew Childs (IQC), Robin Kothari (IQC), and Maris Ozols (Cambridge)

Brendan Rooney

Computational complexity of maxclique for Cayley graphs

Abstract: Maxclique is the problem of finding the largest complete subgraph of an input graph. It is well-known to be an NP-Hard problem for general graphs. In 1998, Codenotti et al. showed that if we restrict to the class of circulant graphs, Maxclique is NP-Hard.

We show that Maxclique is NP-Hard for the class of Cayley graphs on the direct powers of any fixed finite group G. Our proof involves free Cayley graphs, a Goppa code, and quotient graphs.

Aidan Roy

A practical heuristic for finding graph minors

Abstract: We present a heuristic algorithm for finding a small graph H as a minor in a large graph G that is practical for sparse G and H with hundreds of vertices. We also explain the practical importance of finding graph minors in mapping quadratic pseudo-boolean optimization problems onto an adiabatic quantum annealer. This is joint work with Jun Cai and William Macready.

Gordon Royle

Chromatic roots of graphs and matroids

Abstract: The location of the real and/or complex roots of the chromatic polynomial of a graph has been studied for many years, both by combinatorial mathematicians and statistical physicists, yet despite this many fundamental questions remain unsolved. And even though the chromatic polynomial is most generally a matroidal concept, very little indeed is known about the location of chromatic roots of matroids that are not graphic or cographic. In this talk, I will present a necessarily-personal survey of the major results and my favourite open problems in this area.

Simone Severini

From sandwiches to club sandwiches via quantum ideas

Abstract: "The sandwich theorem" is an expression used by Donald Knuth [Electron. J. Combin. 1 (1994), #A1] to describe the behaviour of the Lovasz number of a graph: given by an SDP, it is sandwiched between the clique and the chromatic number, which are both hard. In the last few years we started pupulating the region around the Lovasz number with a bestiary of parameters motivated by quantum mechanical interpretations. Chris Godsil is also not free from responsibility.

Christino Tamon

Quantum walk, state transfer, and complex graphs

Abstract: Given a graph G with adjacency matrix A, a continuous-time quantum walk on G is defined by the unitary matrix $U(t) = exp(-itA)$, as t varies over the real line. Although the original motivation of quantum walk was algorithmic, our focus will be purely graph-theoretic. We say G has state transfer between vertices u and v at time t if the magnitude of the (u, v)-entry of $U(t)$ is equal or close to one. This notion was studied in the context of information transfer in quantum spin chains. From a graph-theoretic view, a main question is to characterize graphs which admit such a state transfer property. We describe some of our results on state transfer on graphs that bear the unmistakable influence of Chris Godsil.

Qing Xiang

The Smith and critical groups of a graph

Abstract: Let G be a finite graph and A its adjacency matrix. The Laplacian matrix of G is defined by $L := D - A$, where D is the diagonal matrix of degrees. Associated with G are two abelian groups. The first is the Smith group $S(G)$ and the second the critical group $K(G)$. We will talk about these groups, with emphasis on the critical group. In particular, we will discuss the recent computations of the Smith and critical group of the Paley graph (in joint work with David Chandler and Peter Sin) using representation theory and number theory.
