Non-Uniform Graph Partitioning

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Problem Definition

Non-Uniform Graph Partitioning

Input:

- $G = (V, E)$, $w : E \to \mathcal{R}_+$.
- Capacities $n_1, n_2, \ldots, n_k$ s.t. $\sum_{j=1}^{k} n_j \geqslant n$.

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- A partition $S_1, \ldots, S_k$ of V where each $|S_j| \leqslant n_j$ minimizing:

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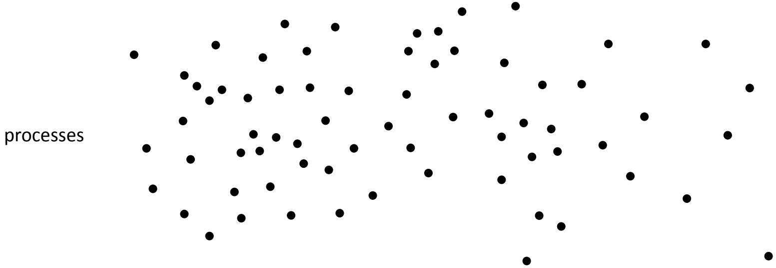
Note:

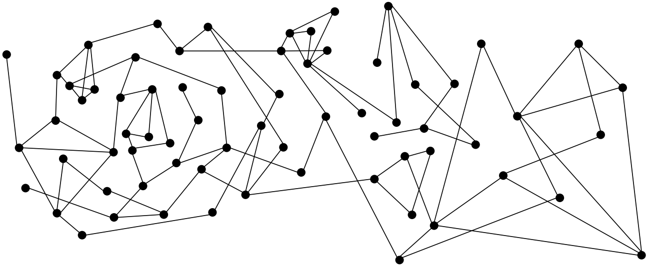
- Number of parts k might depend on n.
- Capacities might be of different magnitudes.

Example - Cloud Computing



Example - Cloud Computing

processes



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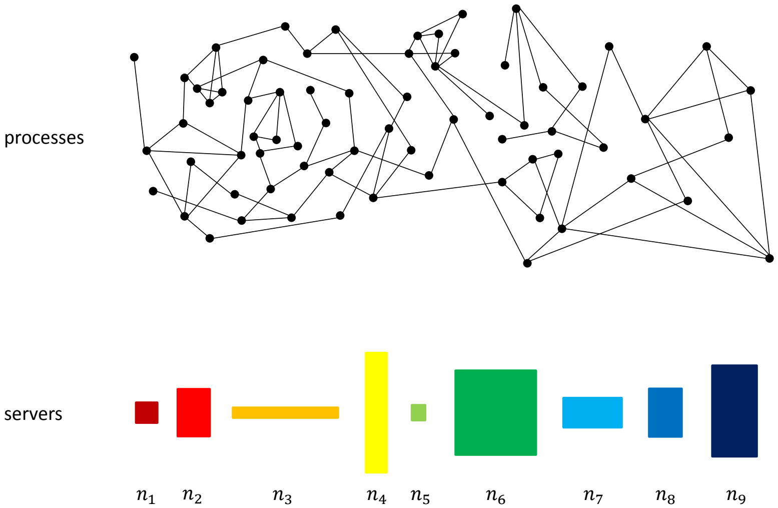

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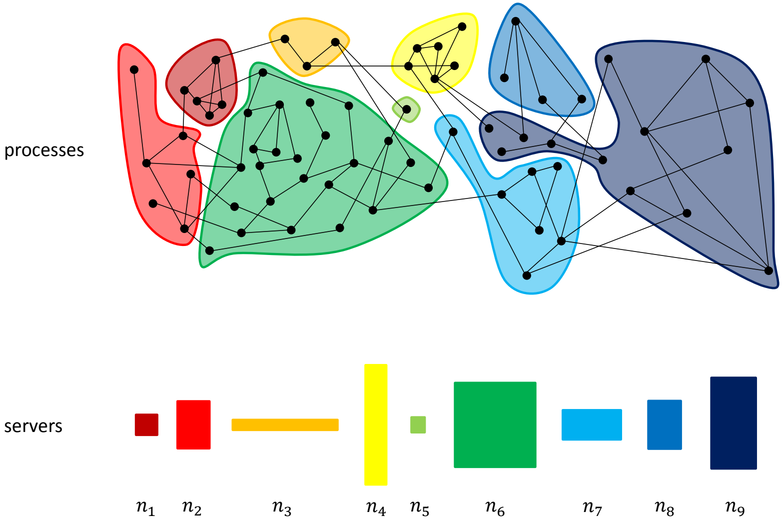

Motivation

Theoretical:

- Captures well studied problems:
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Practical:

- **Cloud and Parallel Computing:** parallelism.
- **Hardware design:** VLSI layout, circuit testing.
- **Data mining:** clustering.
- **Social network analysis:** community discovery.
- **Vision:** pattern recognition.
- **Scientific Computing:** linear systems.

Related Work

Heuristics:

[Barnes-82], [Barnes-Vanneli-Walker-88], [Sanchis-89], [Hadley-Mark-Vanneli-92], [Rendl-Wolkowicz-95] ...

Mainly use spectral theory, local search and quadratic programming.

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Worst Case Guarantee

No meaningful known bounds.

Related Work (Cont.)

Balanced Graph Partitioning: $(n_j \equiv {}^n\!/_k)$

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- Min-Bisection: $(k = 2)$

True Approx. $\begin{cases} O(\log^{3/2} n) & \text{[Feige-Krauthgamer-02]} \\ O(\log n) & \text{[Räcke-08]} \end{cases}$

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Related to Sparsest-Cut and Min b-Balanced-Cut.

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Bicriteria Approx.

$\begin{cases} (O(\log n), 2) & \text{[Even-Naor-Rao-Schieber-99]} \\ (O(\sqrt{\log n \log k}), 2) & \text{[Krauthgamer-Naor-S-09]} \end{cases}$

$\begin{cases} (O(\varepsilon^{-2} \log^{3/2} n), 1+\varepsilon) & \text{[Andreev-Räcke-06]} \\ (O(\log n), 1+\varepsilon) & \text{[Feldmann-Forschini-12]} \end{cases}$

Related Work (Cont.)

Capacitated Metric Labeling:

- The same as Non-Uniform Graph Partitioning with additional assignment costs.

$(O(\log n), O((\log k)))$	$n_j \equiv n/k$	[Naor-S-05]
$(O(\log n), 1)$	constant k	[Andrews-Hajiaghayi-Karloff-Moitra-11]

- $NP \not\subseteq ZPTIME\left(n^{polylog(n)}\right) \implies$
  No finite approximation that violates capacities by $O(\log^{1/2-\varepsilon} k)$, $\forall \varepsilon > 0$.
  [Andrews-Hajiaghayi-Karloff-Moitra-11]

Our Result

Theorem [Krauthgamer-Naor-S-Talwar-13]

There is a bicriteria approximation algorithm achieving a guarantee of:

$$(O(\log n), O(1))$$

for Non-Uniform Graph Partitioning.

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Our Approach

- Configuration LP.
- Randomized rounding $+$ concentration via stopping times.

Configuration LP

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$$\begin{aligned}
(\mathcal{P}) \quad \min \quad & \frac{1}{2}\sum_{j=1}^{k} \sum_{S \in \mathcal{F}_j} \delta(S) \cdot x_{S,j} & \\
s.t. \quad & \sum_{j=1}^{k} \sum_{S \in \mathcal{F}_j : u \in S} x_{S,j} \geqslant 1 & \forall u \in V \\
& \sum_{S \in \mathcal{F}_j} x_{S,j} \leqslant 1 & \forall j = 1, \ldots, k \\
& x_{S,j} \geqslant 0 & \forall j = 1, \ldots, k, \forall S \in \mathcal{F}_j
\end{aligned}$$

Configuration LP (Cont.)

Theorem

$(\mathcal{P})$ can be efficiently solved up to a loss of $O(\log n)$ in the objective.

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- Dual separation oracle of $(\mathcal{P})$ relates to Min ρ-Unbalanced-Cut. Techniques from [Räcke-08] give an $O(\log n)$ approximation.

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  Solution: Scaling constraints differently. ■

Randomized Rounding

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Notation:

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$$W_i \triangleq \left\{ j \ : \ n_j = 2^{-(i-1)} n_1 \right\} \qquad i = 1, 2, \ldots, \ell$$

$$k_i \triangleq |W_i|$$

Randomized Rounding (Cont.)

Idea: Covering vertices by random cuts.

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Rounding - General Approach

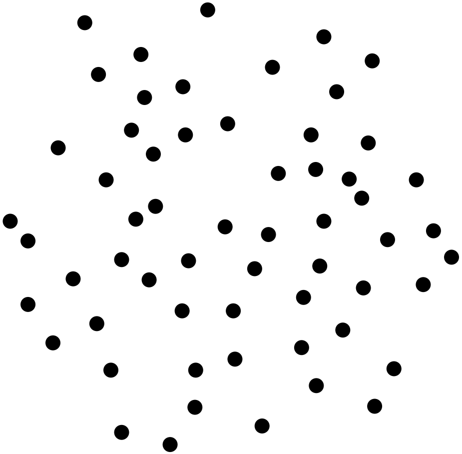
1. $H_i \leftarrow \emptyset$ for every $i = 1, \ldots, \ell$.
2. While $V \neq \emptyset$:
    - Choose $j \sim Unif[1, \ldots, k]$.
    - Choose $S \in \mathcal{F}_j$ w.p. $x_{S,j}$.
    - Let r be the mega-bucket s.t. $j \in W_r$.
    - $H_r \leftarrow H_r \cup \{S \cap V\}$.
    - $V \leftarrow V \setminus S$.

Randomized Rounding - Example

- $H_1 = \phi$
- $H_2 = \phi$
- $H_3 = \phi$

$\vdots$

- $H_\ell = \phi$



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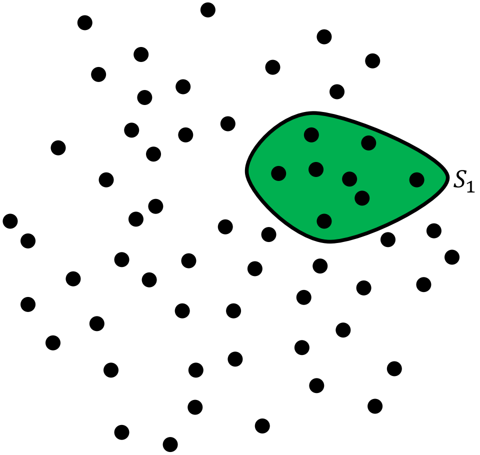

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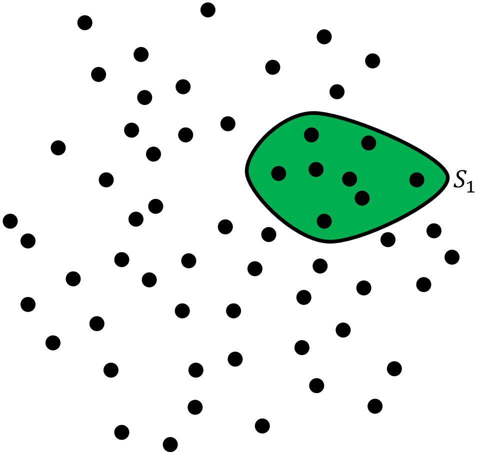

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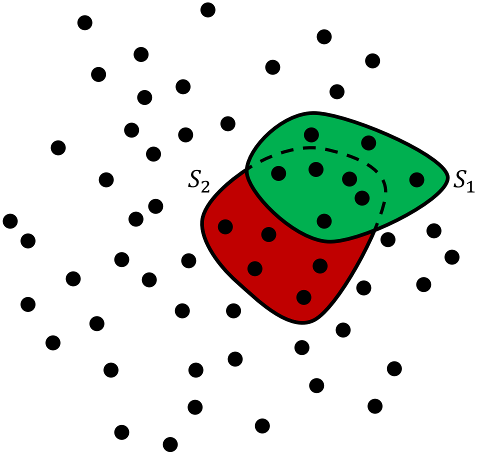

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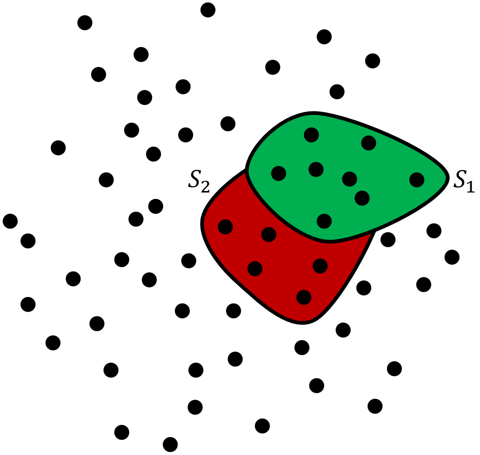

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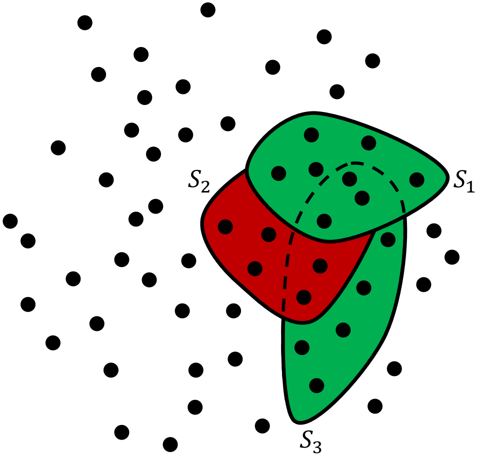

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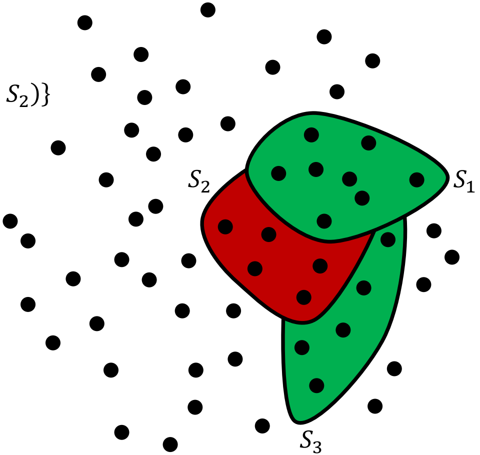

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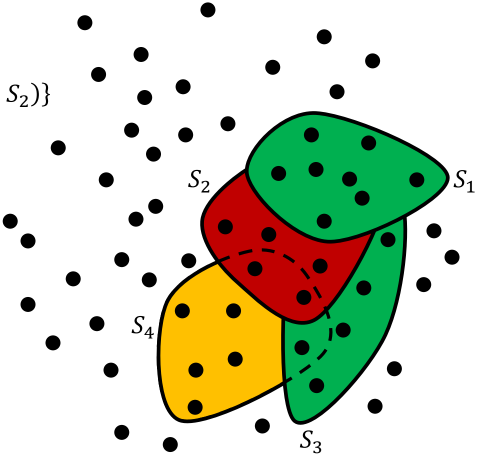

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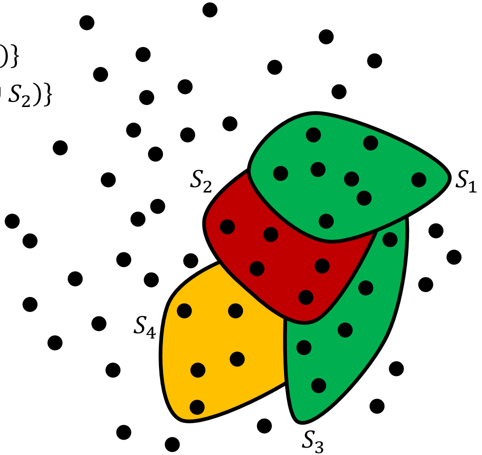

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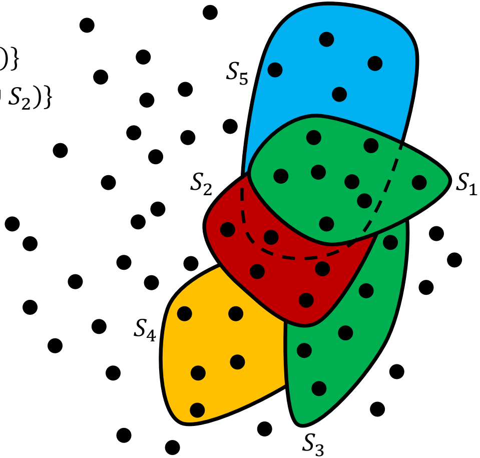

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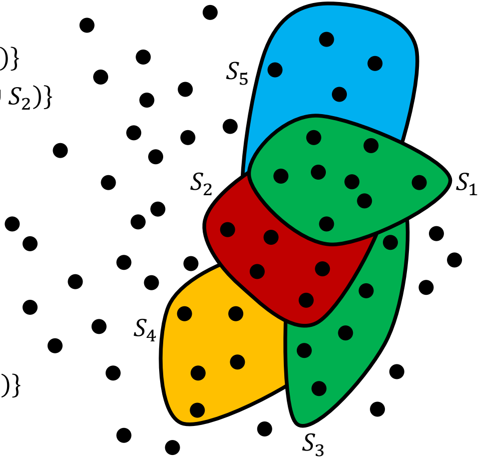

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Question: What to do when all vertices are covered?

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Merge: While $|H_i| > k_i$ merge smallest two cuts in H_i.

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Theorem - Cost Analysis

$$\mathbb{E}\left[cost(ALG)\right] \leqslant 2 \cdot cost\left(\mathcal{P}\right) \ .$$

Proof Outline:

$$\Pr\left[u \text{ and } v \text{ covered in different iterations}\right] \leqslant \sum_{j=1}^{k} \sum_{S \in \mathcal{F}_j : (u,v) \in \delta(S)} x_{S,j} \ .$$

∎

Capacity Analysis - Attempt I

Observation - Interchanging Cuts Within W_i

It suffices to upper bound:

$$N_i \triangleq \text{ number of vertices covered by } H_i \text{ at the end }.$$

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Conclusion: Markov $+$ union bound $\Rightarrow$ $O(\log k)$ capacity violation.

Capacity Analysis - Attempt II

Martingale

$$M_{i,t} \triangleq \mathbb{E}\left[N_i \mid (S_1, j_1), \ldots, (S_t, j_t)\right]$$

$\{M_{i,t}\}_{t=0}^{\infty}$ is a martingale with respect to $\{(S_t, j_t)\}_{t=1}^{\infty}$.

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Conclusion: Azuma + union bound $\Rightarrow$ $O(\sqrt{k \log n \log \log k})$ capacity violation.

Capacity Analysis - Attempt III

Worst Conditional Variance:

Let $(T_1, r_1), \ldots, (T_{t-1}, r_{t-1})$ be the realization that maximizes:

$$\text{Var}\left[M_{i,t} - M_{i,t-1} | (S_1, j_1) = (T_1, r_1), \ldots, (S_{t-1}, j_{t-1}) = (T_{t-1}, r_{t-1})\right] .$$

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$v_{i,t}$ is the worst variance value.

Note:

- For every t a different conditioning might be chosen.
- Martingale concentration via bounded variances (Bernstein) is not sufficient:

  $\sum_{t \geqslant 1} v_{i,t}$ might be too big!

Capacity Analysis - Attempt III (Cont.)

$$\Pr\left[\sum_{t\geqslant 1} \text{Var}\left[M_{i,t} - M_{i,t-1} | (S_1, j_1), \ldots, (S_{t-1}, j_{t-1})\right] \text{ is small}\right] \geqslant \text{ ?}$$

Capacity Analysis - Attempt III (Cont.)

$$\Pr\left[\sum_{t\geqslant 1}\mathsf{Var}\big[M_{i,t}-M_{i,t-1}|(S_1,j_1),\ldots,(S_{t-1},j_{t-1})\big]\ \text{is small}\right]\geqslant\ ?$$

Theorem - Conditional Variances Sum

$$\Pr\left[\sum_{t\geqslant 1}\mathsf{Var}\big[M_{i,t}-M_{i,t-1}\mid(S_1,j_1),\ldots,(S_{t-1},j_{t-1})\big]\geqslant 2\alpha\cdot k_i\cdot 2^{-(i-1)}n_1^2\right]\leqslant {}^1\!/_\alpha\ .$$

Capacity Analysis - Attempt III (Cont.)

$$\Pr\left[\sum_{t\geqslant 1} \mathsf{Var}\left[M_{i,t} - M_{i,t-1} | (S_1, j_1), \ldots, (S_{t-1}, j_{t-1})\right] \text{ is small}\right] \geqslant \text{ ?}$$

Theorem - Conditional Variances Sum

$$\Pr\left[\sum_{t\geqslant 1} \mathsf{Var}\left[M_{i,t} - M_{i,t-1} \mid (S_1, j_1), \ldots, (S_{t-1}, j_{t-1})\right] \geqslant 2\alpha \cdot k_i \cdot 2^{-(i-1)} n_1^2\right] \leqslant {}^1\!/\!_\alpha \ .$$

Intuition: In later iterations the changes are smaller in expectation. ■

Capacity Analysis - Attempt III (Cont.)

Martingale

$$\{M_{i,t}\}_{t=0}^{\infty}$$

Good Event

Variances are small.

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$\searrow \quad \swarrow$

???

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Freedman's Inequality

(stopping-time based concentration)

Capacity Analysis - Attempt III (Cont.)

Immediate Conclusion:
Freedman's inequality yields $O(\log \log k)$ capacity violation.

Capacity Analysis - Attempt III (Cont.)

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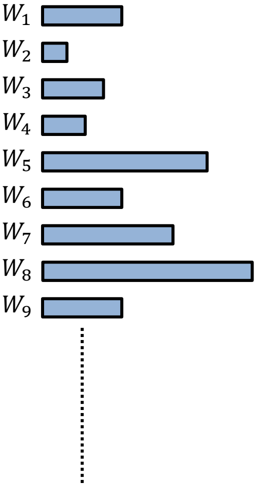
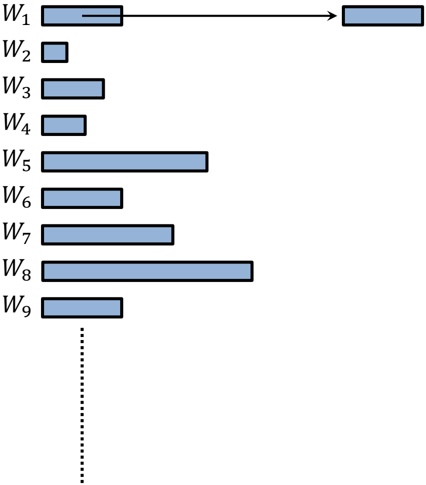
Question

How do we get $O(1)$ capacity violation?

Instance Transformation



Instance Transformation



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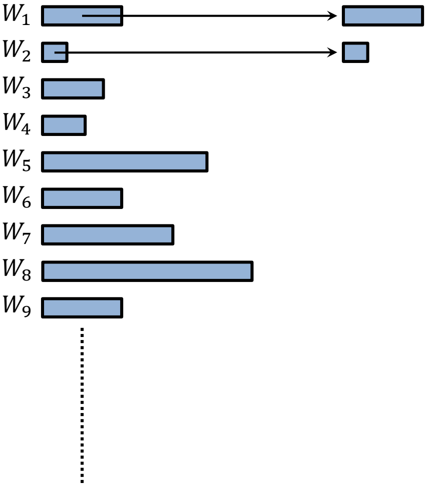

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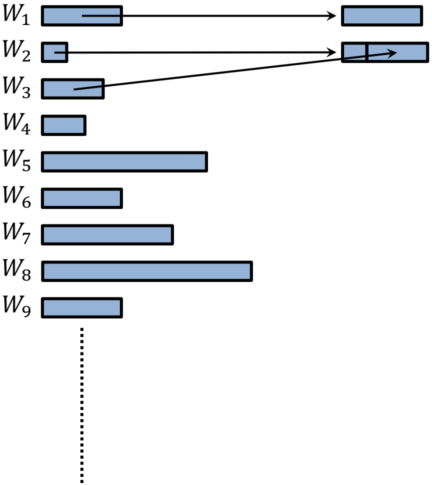

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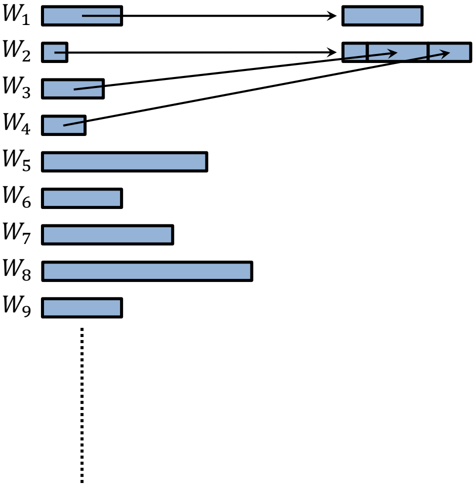

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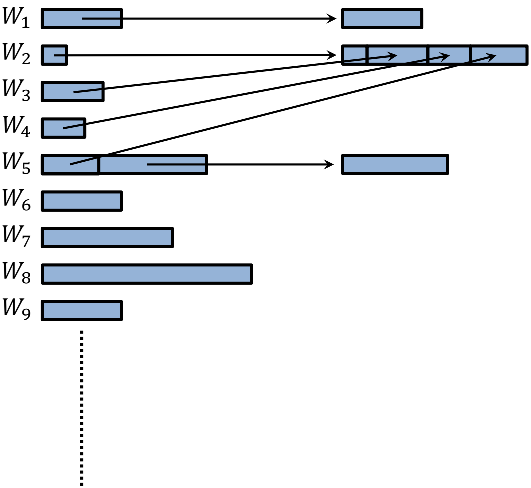

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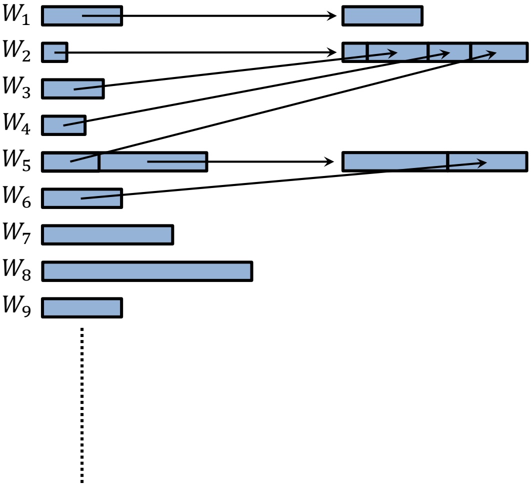

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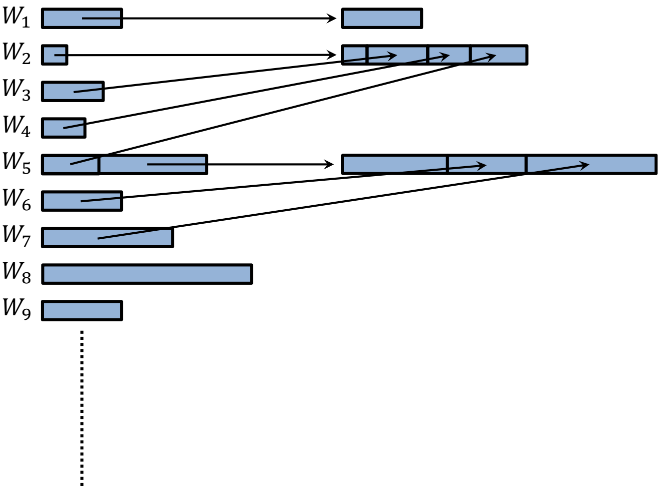

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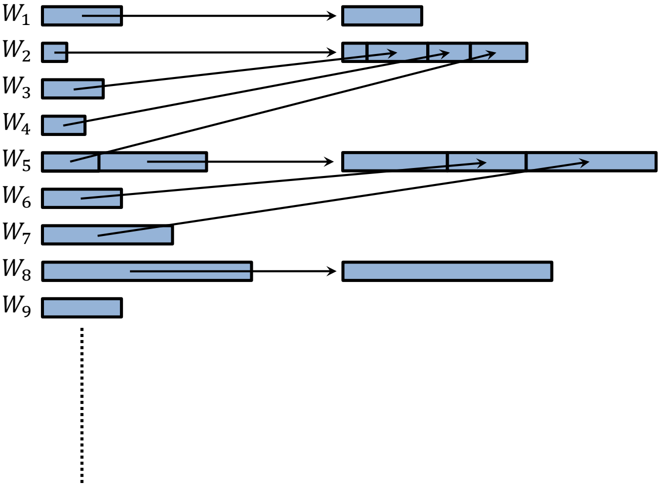

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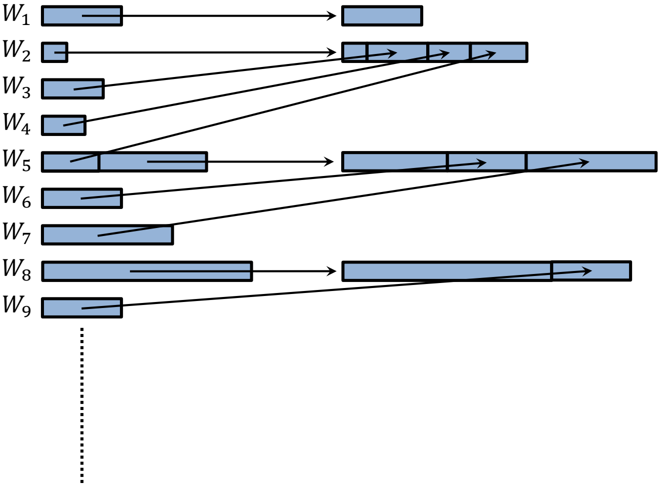

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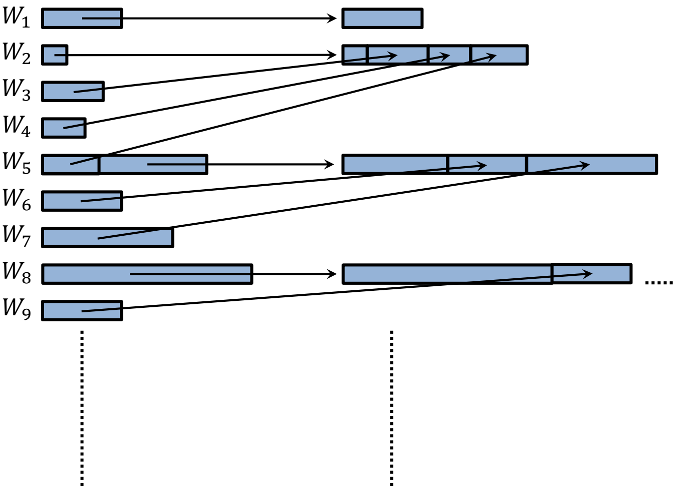

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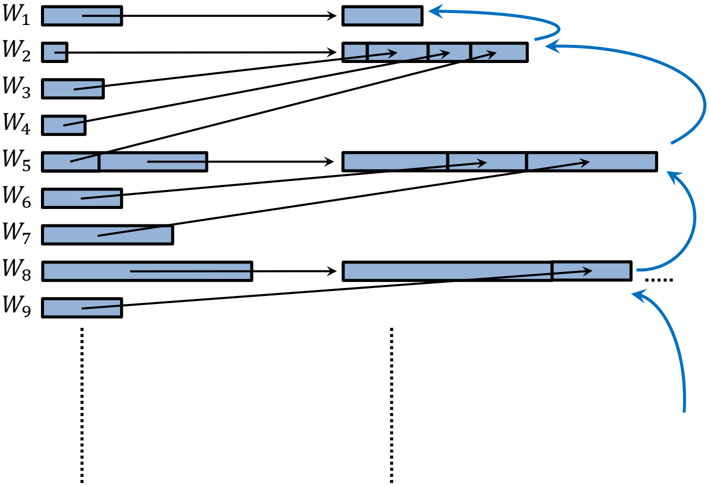

Instance Transformation (Cont.)

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- Total capacity of non-empty W_i grows by a constant c.
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Conclusion: $O(1)$ capacity violation.

Thank You!

Questions?