Master equations of the correlation functions for tensorial group field theory

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Outline

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1. Introduction : motivations
2. Tensorial group fields theory : The model
3. Master equations of correlators of rank 3 and 4 TGFT
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Motivations

1. One of the main purpose of a field theory is to find the exact value of the Green's functions also called correlation functions. Obviously, this can be a highly nontrivial task. In almost scarce cases where this is successfully done, one calls the model exactly solvable.
2. First solvable model : Thirring model, which described self interaction of Dirac field in $1+1$-dimensions. But the famous exactly solvable model is the Schwinger model or QED in 2 dimensions.
3. In a recent work, the renormalizable noncommutative scalar field theory called the Grosse-Wulkenhaar model was solved arXiv :hep-th/0401128 and arXiv :hep-th/0307017. This particular noncommutative field theory projects on a matrix model and then can be seen a model for QG in $2D$.
4. In arXiv :hep-th/0501036 and arXiv :hep-th/0512271 a new proof of the renormalizability was given in direct space using multiscale analysis. The GW propagator breaks the $U(N)$ symmetry invariance in the infrared regime, but is asymptotically safe in the ultraviolet regime arXiv :hep-th/0402093, arXiv :hep-th/0610224 and arXiv :hep-th/0612251.

Introduction : motivations

Motivations

1. In a recent remarkable contribution, Grosse and Wulkenhaar solve successfully all correlators in this model. Using both Ward-Takahashi identities and the Schwinger-Dyson equation, these authors provide, via Hilbert transform, a nonlinear integral equation for the two-point functions arXiv :0909.1389 and arXiv :1205.0465. From this result, they were able to generate solutions for all correlators. Thus, the GW model is exactly nonperturbatively solvable.
2. The question is whether or not this method may apply to other models, in particular to TGFTs dealing with higher rank tensors. We give a partial positive answer of this question. Indeed, as we will show in the following, the resolution method can be applied to find nonlinear equations for the correlations here as well. Due to the highly nontrivial equations and combinatorics, the full resolution of all correlators deserves more work which should be addressed elsewhere.

Introduction : motivations

Random Tensor Models $(1/N)$-expansion

1. Random Tensor Models arXiv :1311.1461, arXiv :1209.5284, arXiv :1112.5104 extends Matrix Models arXiv :hep-th/9306153 as promising candidates to understand Quantum Gravity in higher dimension, $D \geq 3$. The formulation of such models is based on a Feynman path integral generating randomly graphs representing simplicial pseudo manifolds of dimension D.
2. The equivalent of the t'Hooft large N limit for these Tensor Models has been recently discovered by Razvan Gurau arXiv :1011.2726, arXiv :1101.4182 and arXiv :1102.5759. The large N limit behaviour is a powerful tool which allows to understand the continuous limit of these models through, for instance, the study of critical exponents and phase transitions.

Introduction : motivations

Renormalization of TGFT

1. With the advent of the field theory formulation of Random Tensor Models, henceforth called Tensorial Group Field Theory (TGFT), one addresses several different questions such as Renormalizability (for removing divergences) and the study UV behaviour of these models. It turns out that Renormalization can be consistently defined for TGFTs and most of them, for the higher rank $D \geq 3$ are UV asymptotically free arXiv :1306.1201. This is of course very encouraging for the Geomeogenesis Scenario arXiv :1205.5513, arXiv :1303.7256.
2. It becomes more and more convincing that Random Tensors and TGFT's will take a growing role for giving answers for the Quantum Gravity conundrum. Despite all these results, a lot of questions (both conceptual and technical) arise in this framework for obtaining a final and emergent theory of General Relativity . Among other goals, it would be strongly desirable to establish more connections with other studies and important results around Gravity.

Introduction : motivations

Main goal of this presentation

The purpose of this presentation is to provide the first glimpses of the extension of the recent full resolution of the correlation functions in the Grosse-Wulkenhaar model, by using both Ward-Takahashi identities and Schwinger-Dyson equation.

The model

TGFT on $U(1)^D$

We consider here TGFT, which is defined by an action $S[\bar{\varphi},\varphi]$:

$$S[\bar{\varphi},\varphi]=\sum_{p_i}\bar{\varphi}_{12\cdots D}C^{-1}(p_1,p_2,\cdots,p_D;p'_1,p'_2,\cdots p'_D)\varphi_{12\cdots D}\prod_{i=1}^{D}\delta_{p_ip'_i}+S^{\text{int}}. \quad (1)$$

φ and its conjugate $\bar{\varphi}$ are defined on the compact Lie group $G=U(1)$ i.e.

$$\varphi: G^D\longrightarrow\mathbb{C};\ \ (g_1,\cdots,g_D)\longmapsto\varphi(g_1,\cdots,g_D). \quad (2)$$

We are using the Fourier transformation of the field. The momentum variable associated of $[g]=(g_1,g_2,\cdots,g_D)\in U(1)^D$ is $[p]=(p_1,p_2,\cdots,p_D)\in\mathbb{Z}^D$. Using the parametrization $g_k=e^{i\theta_k}$ and $\varphi_{12\cdots D}=:\varphi(p_1,\cdots,p_D)=:\varphi_{[D]}$

$$\varphi(g_1,\cdots,g_D)=\sum_{p_i\in\mathbb{Z}}\varphi(p_1,\cdots,p_D)e^{i\sum_k\theta_kp_k},\quad \theta_i\in[0,2\pi). \quad (3)$$

The model

Invariance

The field φ and its conjugate $\bar{\varphi}$, are transformed under the tensor product of D fundamental representations of the unitary group $\mathcal{U}_{\otimes}^{N_D} := \otimes_{i=1}^{D} U(N_i)$. Let $U^{(a)} \in U(N_a)$, $a = 1, 2, \cdots, D$. The field φ and its conjugate $\bar{\varphi}$ are transformed under $U(N_a)$ as

$$\varphi_{12\cdots D} \to [U^{(a)}\varphi]_{12\cdots a\cdots D} = \sum_{p'_a \in Z} U^{(a)}_{p_a p'_a} \varphi_{12\cdots a'\cdots D}, \quad (4)$$

$$\bar{\varphi}_{12\cdots D} \to [\bar{\varphi} U^{\dagger(a)}]_{12\cdots a\cdots D} = \sum_{p'_a \in Z} \bar{U}^{(a)}_{p_a p'_a} \bar{\varphi}_{12\cdots a'\cdots D}. \quad (5)$$

p'_a or simply a' is the momentum index at the position a in the expression $\varphi_{12\cdots a'\cdots D}$. The kinetic action is re-expressed as follows

$$S^{\text{kin}}[\bar{\varphi}, \varphi] = \sum_{p_1, \cdots, p_D} \varphi_{12\cdots D} M_{12\cdots D} \bar{\varphi}_{12\cdots D}, \quad M_{12\cdots D} = C^{-1}_{12\cdots D}. \quad (6)$$

The Model

Variation of the action

The variation of the action S^{kin} under infinitesimal $U(N_a)$ transformation is given by

$$\delta^{(a)}[S^{\text{kin}}] = -i \sum_{p_1,\cdots,p_D} \Big[M\Big(\varphi[\bar{B}\bar{\varphi}]^{(a)} - [B\varphi]^{(a)}\bar{\varphi}\Big)\Big]_{12\cdots D} \tag{7}$$

where B is the infinitesimal Hermitian operator corresponding to the generator of unitary group $U(N_a)$ i.e.

$$U^{(a)}_{pp'} = \delta^{(a)}_{pp'} + iB^{(a)}_{pp'} + O(B^2), \quad \bar{U}^{(a)}_{pp'} = \delta^{(a)}_{pp'} - i\bar{B}^{(a)}_{pp'} + O(\bar{B}^2), \tag{8}$$

with $\bar{B}^{(a)}_{pp'} = B^{(a)}_{p'p}$.

The model

Partition function

Consider now the theory defined with external source $F[\varphi, \bar{\varphi}; \eta, \bar{\eta}]$ as

$$F[\eta, \bar{\eta}] = \sum_{p_1, \cdots, p_D} \bar{\varphi}_{12\cdots D}\eta_{12\cdots D} + \bar{\eta}_{12\cdots D}\varphi_{12\cdots D}. \tag{9}$$

The partition function of the model is re-expressed as

$$\mathcal{Z}[\eta, \bar{\eta}] = \int d\varphi d\bar{\varphi}\, e^{-S[\varphi,\bar{\varphi}]+F[\varphi,\bar{\varphi};\eta,\bar{\eta}]}. \tag{10}$$

Under $U(N_a)$ infinitesimal transformation

$$\delta^{(a)}[F] = i \sum_{p_1, \cdots, p_D} \Big[\bar{\eta}[B\varphi]^{(a)} - [\bar{B}\bar{\varphi}]^{(a)}\eta\Big]_{12\cdots D}. \tag{11}$$

The model

Total variation

Let $\delta^{(\otimes)}$ be the total variation under the action of the group element $U^{(1)} \otimes U^{(2)} \otimes \cdots \otimes U^{(D)} \in \mathcal{U}_{\otimes}^{N_D}$. Then we get the following result : The kinetic term of the action, i.e. S^{kin} and F are respectively transformed linearly as

$$\delta^{(\otimes)} S^{\text{kin}} = \sum_{a=1}^{D} \delta^{(a)} S^{\text{kin}} , \quad \delta^{(\otimes)} F = \sum_{a=1}^{D} \delta^{(a)} F. \tag{12}$$

Then

$$\delta^{(\otimes)} S = 0 \longleftrightarrow \delta^{(a)} S = 0, \quad \forall\ S = S[\varphi, \bar{\varphi}, \eta, \bar{\eta}]. \tag{13}$$

We assume that $N_i = N$, $i = 1, 2 \cdots D$, and we take the interaction terms such that there are invariant under the transformation $U^{(a)}$ i.e. $\delta^{(a)} S^{\text{int}} = 0$. This is the new input in TGFT's : the $U(N)$ tensor invariance must be the one defining the interaction.

Ward-Takahashi identities

Proof of proposition 1

The ward Takahashi identities for $U(1)^d$ TGFT without gauge invariant condition is given by

$$\sum_{[p]}\Big(M_m - M_n\Big)\langle\Big[\frac{\partial(\bar\eta\varphi)}{\partial\bar\eta}\frac{\partial(\bar\varphi\eta)}{\partial\eta}\Big]\varphi_n\bar\varphi_m\rangle_c$$
$$=\sum_{[p]}\langle\frac{\partial(\bar\eta_m\varphi_n)}{\partial\bar\eta}\frac{\partial(\bar\varphi\eta)}{\partial\eta}\rangle_c-\sum_{[p]}\langle\frac{\partial(\bar\varphi_m\eta_n)}{\partial\eta}\frac{\partial(\bar\eta\varphi)}{\partial\bar\eta}\rangle_c. \quad (14)$$

Note that the equation (14) is valid for all positions indices $a = 1, 2, \cdots, D$. Let us also remark that for $m = n$ the left hand side (lhs) of the equation (14) vanishes. In the double derivative $\partial_{\bar\eta}\partial_\eta$, we fix the indices such that $\bar\eta_{[\alpha]}\eta_{[\beta]}$.

Partial conclusion and remark

- In conclusion, there are exactly D Ward-Takahashi identities for the rank D TGFT's associated with this type of invariance.
- Furthermore, we mention that we are not considering the TGFT with gauge invariance condition. We consider here the simplest the TGFT as treated in work of Joseph Ben Geloun et al.
- Most of the result of this work might be extended to this different framework with not much work since only the propagator will be modified.
- Thus, one expects similar Ward identities in that gauge invariant framework .

Rank 3 TGFT on $U(1)$

The model

The renormalizable $3D$ tensor model is defined by the action S_{3D}, in which the kinetic term take's the form

$$S_{3D}^{\text{kin}} = \sum_{[p]} \bar{\varphi}_{123}\, C_{123}^{-1}\, \varphi_{123}, \tag{15}$$

where

$$C_{abc} = Z^{-1}(|a| + |b| + |c| + m^2)^{-1}, \quad a, b, c \in \mathbb{Z}. \tag{16}$$

The field strength can be modified as follows :

$$\varphi \longrightarrow (Z_1 Z_2 Z_3)^{\frac{1}{6}}\, \varphi = Z^{1/2}\varphi, \quad Z_\rho = 1 - \partial_{b_\rho} \Gamma_{b_1 b_2 b_3}\Big|_{b_{1,2,3}=0}, \tag{17}$$

where $\Gamma_{b_1 b_2 b_3}$ is the self-energy or one particle irreducible (1PI) two-point functions.

Rank 3 TGFT on $U(1)$

The model

The interaction of the model is defined by the three contributions V_1, V_2, and V_3 expressed in momentum space as

$$\begin{aligned} S^{\text{int}}_{3D} &= \lambda_1 Z^2 \sum_{\substack{1,2,3\\1',2',3'}} \varphi_{123}\bar\varphi_{321'}\varphi_{1'2'3'}\bar\varphi_{3'2'1} + \lambda_2 Z^2 \sum_{\substack{1,2,3\\1',2',3'}} \varphi_{123}\bar\varphi_{32'1}\varphi_{1'2'3'}\bar\varphi_{3'21'} \\ &+ \lambda_3 Z^2 \sum_{\substack{1,2,3\\1',2'3'}} \varphi_{123}\bar\varphi_{3'21}\varphi_{1'2'3'}\bar\varphi_{32'1'} = \lambda_1 V_1 + \lambda_2 V_2 + \lambda_3 V_3, \end{aligned} \quad (18)$$

and are represented as

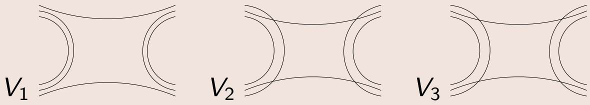


FIGURE: The vertices of rank 3 tensor model

Rank 3 TGFT on $U(1)$

Ward-Takahashi identities

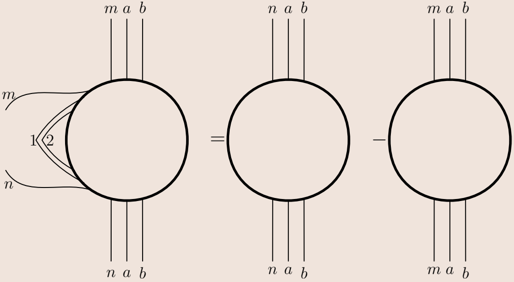


FIGURE: Ward-Takahashi identities

The Schwinger-Dyson equation of two-point functions

$\Gamma_{abc} =$

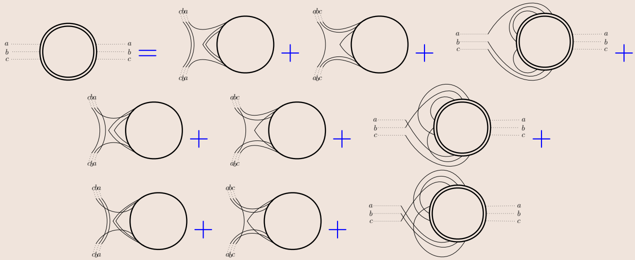


FIGURE: Schwinger-Dyson equation for 1PI two-point functions

In this figure the quantity Γ_{abc} is the self-energy or 1PI two-point functions, which can be written as

$$\Gamma_{abc} = \sum_{\rho=1}^{3} \Gamma^{\rho}_{abc}, \quad \text{where} \quad \Gamma^{\rho}_{abc} = T^{\rho}_{abc} + \Sigma^{\rho}_{abc}. \tag{19}$$

How to combine Ward identity and Schwinger-Dyson equation

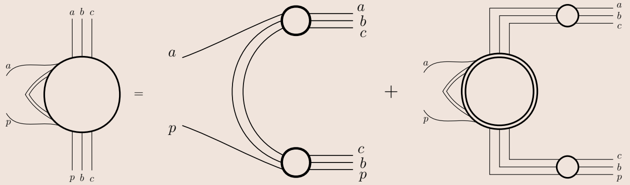


FIGURE: Decomposition of the two-point functions with insertion : Case where $\rho = 1$

Using This decomposition we get

$$\Sigma^1_{abc} = Z^2\lambda_1 \sum_p G^{-1}_{abc} G^{ins}_{[ap]bc}, \quad T^1_{abc} = Z^2\lambda_1 \sum_{p,q} G_{apq}. \tag{20}$$

How to combine Ward identity and Schwinger-Dyson equation

In the same manner we can obtain the following decomposition :

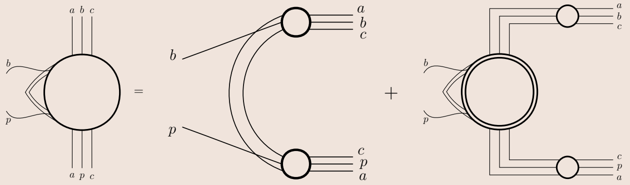


FIGURE: Decomposition of the two-point functions with insertion : Case where $\rho = 2$

which allows to obtain the relation

$$\Sigma^2_{abc} = Z^2\lambda_2 \sum_p G^{-1}_{abc} G^{ins}_{[bp]ca}, \quad T^2_{abc} = Z^2\lambda_2 \sum_{p,q} G_{pbq} \qquad (21)$$

How to combine Ward identity and Schwinger-Dyson equation

Also we get the following decomposition :

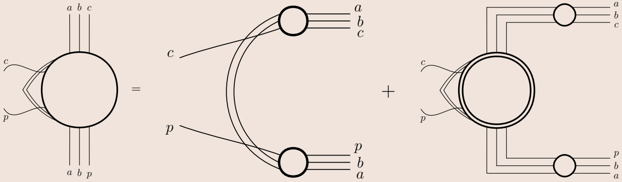


FIGURE: Decomposition of the two-point functions with insertion : Case where $\rho = 3$

$$\Sigma^3_{abc} = Z^2\lambda_3 \sum_p G^{-1}_{abc} G^{ins}_{[cp]ab}, \quad T^3_{abc} = Z^2\lambda_3 \sum_{p,q} G_{pqc}. \tag{22}$$

How to combine Ward identity and Schwinger-Dyson equation

Therefore using the last expressions (20), (21) and (22), the 1PI two-point functions take the form

$$\begin{aligned}\Gamma_{abc} &= Z^2\lambda_1\sum_{p,q}G_{apq}+Z^2\lambda_2\sum_{p,q}G_{pbq}+Z^2\lambda_3\sum_{p,q}G_{pqc}\\ &+ Z\lambda_1\sum_p G^{-1}_{abc}\frac{G_{abc}-G_{pbc}}{|p|-|a|}+Z\lambda_2\sum_p G^{-1}_{abc}\frac{G_{bca}-G_{pca}}{|p|-|b|}\\ &+ Z\lambda_3\sum_p G^{-1}_{abc}\frac{G_{cab}-G_{pab}}{|p|-|c|}.\end{aligned} \tag{23}$$

We assume now that the function G_{abc} satisfy the condition

$$G_{abc}=G_{bca}=G_{cab} \tag{24}$$

Proposition 2

Symmetry properties : The connected two-point functions Γ^2_{abc} can be obtained using Γ^1_{abc} and replace respectively $a \to b$ and $b \to c$ and $c \to a$. In the same manner Γ^3_{abc} can be obtained using Γ^1_{abc} and replacing respectively $a \to c$, $b \to a$ and $c \to b$.

Now using the relation $G^{-1}_{abc} = M_{abc} - \Gamma_{abc}$, we get

$$\Gamma^1_{abc} = Z^2\lambda_1\Big[\sum_{pq}\frac{1}{M_{apq}-\Gamma_{apq}}+\sum_{p}\frac{1}{M_{pbc}-\Gamma_{pbc}}-\sum_{p}\frac{1}{M_{pbc}-\Gamma_{pbc}}\frac{\Gamma_{abc}-\Gamma_{pbc}}{Z(|a|-|p|)}\Big],$$

$$\Gamma^2_{abc} = Z^2\lambda_2\Big[\sum_{pq}\frac{1}{M_{pbq}-\Gamma_{pbq}}+\sum_{p}\frac{1}{M_{pca}-\Gamma_{pca}}-\sum_{p}\frac{1}{M_{pca}-\Gamma_{pca}}\frac{\Gamma_{bca}-\Gamma_{pca}}{Z(|b|-|p|)}\Big],$$

$$\Gamma^3_{abc} = Z^2\lambda_3\Big[\sum_{pq}\frac{1}{M_{pqc}-\Gamma_{pqc}}+\sum_{p}\frac{1}{M_{pab}-\Gamma_{pab}}-\sum_{p}\frac{1}{M_{pab}-\Gamma_{pab}}\frac{\Gamma_{cab}-\Gamma_{pab}}{Z(|c|-|p|)}\Big].$$

Renormalized self-energy

For the rest of this section we consider the connected two-point functions Γ^1_{abc} and finally Γ^2_{abc} and Γ^3_{abc} will be deduced using the proposition 2. Then we pass to renormalized quantities using the Taylor expansion as

$$\Gamma^1_{abc} = (Z-1)(|a|+|b|+|c|) + Zm^2_{bar} - m^2 + \Gamma^{phys}_{abc}. \tag{25}$$

The equation (23) takes the form (we set $\lambda_1 = \lambda$)

$$\begin{aligned} &Zm^2_{bar} - m^2 + (Z-1)(|a|+|b|+|c|) + \Gamma^{phys}_{abc} \\ &= Z^2\lambda \sum_{p,q} \frac{1}{|p|+|q|+|a|+m^2-\Gamma^{phys}_{pqa}} \\ &+ Z\lambda\Big[\sum_p \frac{1}{|p|+|b|+|c|+m^2-\Gamma^{phys}_{pbc}} \\ &- \frac{1}{|p|+|b|+|c|+m^2-\Gamma^{phys}_{pbc}} \frac{\Gamma^{phys}_{abc}-\Gamma^{phys}_{pbc}}{(|a|-|p|)}\Big]. \end{aligned} \tag{26}$$

Continuous limit

Prosposition 3

The master equation of the two-point functions Γ^{phys}_{abc} is

$$
\begin{aligned}
&(Z-1)(|a|+|b|+|c|)+\Gamma^{phys}_{abc}\\
&=2Z^2\lambda\int_0^\infty |p|d|p|\Big[\frac{1}{2|p|+|a|+m^2-\Gamma^{phys}_{ppa}}-\frac{1}{2|p|+m^2-\Gamma^{phys}_{pp0}}\Big]\\
&+2Z\lambda\int_0^\infty d|p|\Big[\frac{1}{|p|+|b|+|c|+m^2-\Gamma^{phys}_{pbc}}-\frac{1}{|p|+m^2-\Gamma^{phys}_{p00}}\\
&-\frac{1}{|p|+|b|+|c|+m^2-\Gamma^{phys}_{pbc}}\frac{\Gamma^{phys}_{abc}-\Gamma^{phys}_{pbc}}{(|a|-|p|)}+\frac{1}{|p|+m^2-\Gamma^{phys}_{p00}}\frac{\Gamma^{phys}_{p00}}{|p|}\Big] \qquad (27)
\end{aligned}
$$

Change of variables

with $p \in \mathbb{R}^+$. We introduce a change of variables

$$|a| = m^2 \frac{\alpha}{1-\alpha}, \quad |b| = m^2 \frac{\beta}{1-\beta}, \quad |c| = m^2 \frac{\gamma}{1-\gamma}, \quad |p| = m^2 \frac{\rho}{1-\rho}, \tag{28}$$

$$\Gamma^{phys}_{abc} = m^2 \frac{\Gamma_{\alpha\beta\gamma}}{(1-\alpha)(1-\beta)(1-\gamma)}. \tag{29}$$

We also take the cutoff Λ such that $p_\Lambda = m^2 \frac{\Lambda}{1-\Lambda}$. Let us now define the quantity $G_{\alpha\beta\gamma}$ as

$$1 - \alpha\beta - \alpha\gamma - \beta\gamma + 2\alpha\beta\gamma - \Gamma_{\alpha\beta\gamma} = \frac{1 - \alpha\beta - \alpha\gamma - \beta\gamma + 2\alpha\beta\gamma}{G_{\alpha\beta\gamma}}. \tag{30}$$

Closed equation

Now using the symmetry properties between the indices of G_{abc} and all the result above, we come to the following result

Theorem

The closed equation of the symmetric two-point functions $G_{\alpha\beta\gamma}$ satisfies the nonlinear integral equation

$$G_{\alpha\beta\gamma} = 1 + \lambda'\Big\{\mathcal{Y} + \int_0^1 d\rho\, G_{\rho 00} + \frac{(1-\alpha)(1-\beta)(1-\gamma)}{1-\alpha\beta-\alpha\gamma-\beta\gamma+2\alpha\beta\gamma}\Big[\int_0^1 d\rho\Big(\frac{G_{\rho\beta\gamma}}{\alpha-\rho} + \frac{(2\beta\gamma-\beta-\gamma)G_{\rho\beta\gamma}}{1-\beta\rho-\gamma\rho-\gamma\beta+2\rho\gamma\beta}\Big) + G_{\alpha\beta\gamma}\Big[\int_0^1 d\rho\,\frac{G_{\rho\alpha 0}-G_{\rho 00}}{1-\rho} + \int_0^1 d\rho\,\frac{G_{\rho 00}}{\rho} - \mathcal{Y} - \int_0^1 d\rho\, G_{\rho 00} - G_{0\alpha 0}^{-1}\Big(\int_0^1 d\rho\,\frac{G_{\rho\alpha 0}}{\rho} + \alpha\int_0^1 d\rho\,\frac{G_{\rho\alpha 0}}{1-\alpha\rho} + \int_0^1 d\rho\,\frac{G_{\rho 00}}{\alpha-\rho}\Big)\Big]\Big\} \quad (31)$$

where

$$\mathcal{Y} = \lim_{\epsilon\to 0}\int_0^1 d\rho\,\frac{G_{\rho\epsilon 0}-G_{\rho 00}}{\epsilon\rho}, \qquad \lambda' = \frac{2\lambda}{m^2}. \quad (32)$$

Solution

Proposition 3

Pertubatively, at second order, using the Cauchy principal value is given by

$$G_{\alpha\beta\gamma} = 1 + \lambda' \mathcal{X}^{(1)}_{\alpha\beta\gamma} + \lambda'^2 \mathcal{X}^{(2)}_{\alpha\beta\gamma} + O(\lambda'^3), \quad G_{000} = 1 \tag{33}$$

$$\mathcal{X}^{(1)}_{\alpha\beta\gamma} = 1 + \frac{(1-\alpha)(1-\beta)(1-\gamma)}{1-\alpha\beta-\alpha\gamma-\beta\gamma+2\alpha\beta\gamma}\Big(\ln(1-\alpha) - 1 + \ln\frac{\beta\gamma-\beta-\gamma+1}{1-\beta\gamma}\Big),$$

$$\begin{aligned}\mathcal{X}^{(2)}_{\alpha\beta\gamma} &= \frac{\pi^2}{6} - \frac{3}{2} + \frac{(1-\alpha)(1-\beta)(1-\gamma)}{1-\alpha\beta-\alpha\gamma-\beta\gamma+2\alpha\beta\gamma}\Big[\mathcal{X}^{(1)}_{\alpha\beta\gamma}\Big(\ln\frac{(1-\alpha)^2}{\alpha} - 1\Big) \\ &+ \int_0^1 d\rho \frac{(2\beta\gamma-\beta-\gamma)\mathcal{X}^{(1)}_{\rho\beta\gamma}}{1-\beta\rho-\gamma\rho-\beta\gamma+2\beta\gamma\rho} + \int_0^1 d\rho \frac{\mathcal{X}^{(1)}_{\rho\alpha 0} - \mathcal{X}^{(1)}_{\rho 00}}{1-\rho} - \frac{\pi^2}{6} + \frac{3}{2} \\ &- \alpha\int_0^1 d\rho\, \frac{\mathcal{X}^{(1)}_{\rho\alpha 0}}{1-\alpha\rho} - \int_0^1 d\rho \frac{\mathcal{X}^{(1)}_{\rho\alpha 0} - \mathcal{X}^{(1)}_{\rho 00} + \mathcal{X}^{(1)}_{0\alpha 0}}{\rho} - \mathcal{X}^{(1)}_{0\alpha 0}\ln\frac{(1-\alpha)^2}{\alpha}\Big].\end{aligned}$$

Closed equation for two point function of rank 4 TGFT

The same method use in last section will be performed here to establish the renormalized two-point functions of rank 4 tensor field firstly. The action S_{4D} of the model is also subdivided into two terms as

$$S_{4D} = S_{4D}^{\text{kin}} + S_{4D}^{\text{int}}. \tag{34}$$

The kinetic term S_{4D}^{kin} is given by

$$S_{4D}^{\text{kin}} = \sum_{p_j \in \mathbb{Z}} \varphi_{1234} \Big(\sum_{i=1}^{4} p_i^2 + m^2 \Big) \bar{\varphi}_{1234}. \tag{35}$$

Noting that in four dimensional case the renormalization is guaranteed by the presence of the propagator associated with the heat kernel :

$$C([p]) = \Big(\sum_{i=1}^{4} p_i^2 + m^2 \Big)^{-1} = M_{1234}^{-1}. \tag{36}$$

Closed equation for two point function of rank 4 TGFT

S_{4D}^{int} is related to the interaction, which is divided into three fundamental contributions $V_{6,1}$, $V_{6,2}$ and $V_{4,1}$ given by

$$V_{6;1} = \sum_{p_j\in\mathbb{Z}} \varphi_{1234}\,\bar{\varphi}_{1'234}\,\varphi_{1'2'3'4'}\,\bar{\varphi}_{1''2'3'4'}\,\varphi_{1''2''3''4''}\,\bar{\varphi}_{12''3''4''} + \text{permuts} \quad (37)$$

$$V_{6;2} = \sum_{p_j\in\mathbb{Z}} \varphi_{1234}\,\bar{\varphi}_{1'2'3'4}\,\varphi_{1'2'3'4'}\,\bar{\varphi}_{1''234'}\,\varphi_{1''2''3''4''}\,\bar{\varphi}_{12''3''4''} + \text{permuts} \quad (38)$$

$$V_{4;1} = \sum_{p_j\in\mathbb{Z}} \varphi_{1234}\,\bar{\varphi}_{1'234}\,\varphi_{1'2'3'4'}\,\bar{\varphi}_{12'3'4'} + \text{permuts} \quad (39)$$

and an anomalous term, namely $V_{4,2}$

$$V_{4;2} = \Big(\sum_{p_j\in\mathbb{Z}} \bar{\varphi}_{1234}\,\varphi_{1234}\Big)\Big(\sum_{p_j\in\mathbb{Z}} \bar{\varphi}_{1'2'3'4'}\,\varphi_{1'2'3'4'}\Big). \quad (40)$$

Closed equation for rank 4 TGFT

Vertex representation case $\rho = 1$ and $\rho\rho' = 14$

Let us immediately emphasize that the vertices of the type $V_{6,1}$ and $V_{4,1}$ are parametrized by four indices $\rho \in \{1, 2, 3, 4\}$, and the vertices contributing to $V_{6,2}$ are parametrized by six index values $\rho\rho' \in \{1.2, 1.3, 1.4, 2.3, 2.4, 3.4\}$. The couple $\rho\rho'$ will be totally symmetric i.e., $\rho\rho' = \rho'\rho$.

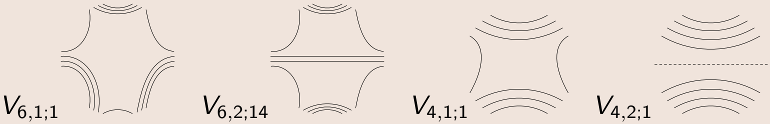
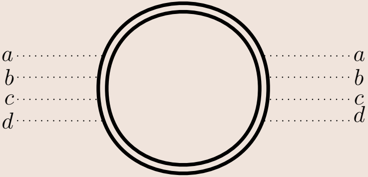


FIGURE: Vertex representation of $4D$ tensor model

Closed equation for rank 4 TGFT

Schwinger-Dyson equation



$$= \Gamma_{abcd} = \sum_{\rho} \left(\Gamma^{6,1;\rho}_{abcd} + \Gamma^{4,1;\rho}_{abcd} \right) + \sum_{\rho\rho'} \Gamma^{6,2;\rho\rho'}_{abcd}$$

FIGURE: Schwinger-Dyson equation of rank 4 tensor model

Closed equation for rank 4 TGFT

Schwinger-Dyson equation

$\Gamma^{6,1}_{abcd} =$

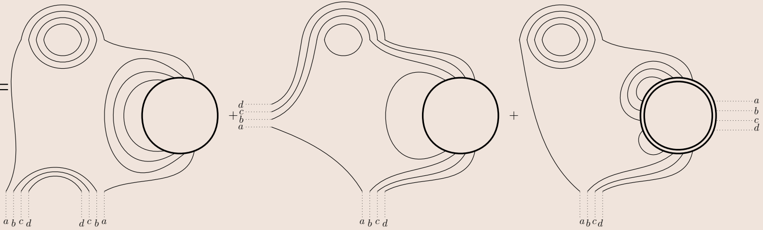


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FIGURE:

Closed equation for rank 4 TGFT

Schwinger-Dyson equation

$\Gamma^{6,2}_{abcd} =$

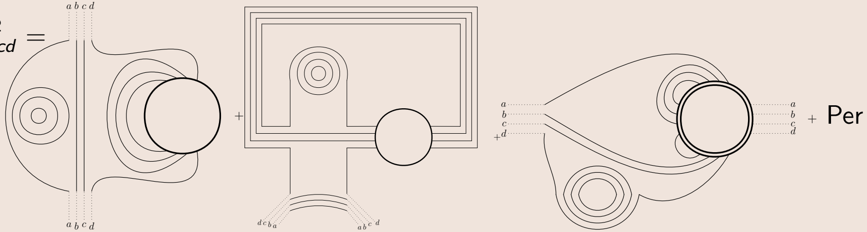


FIGURE:

Closed equation for rank 4 TGFT

Schwinger-Dyson equation

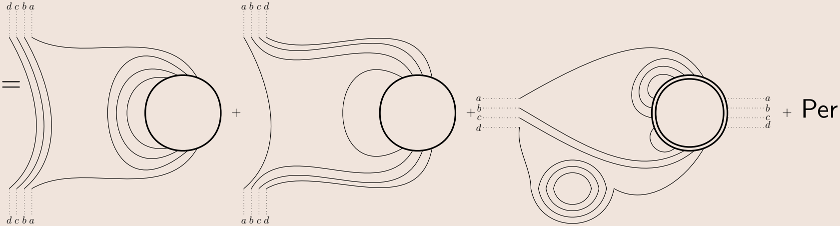
$\Gamma^{4,1}_{abcd} =$ 

FIGURE:

Closed equation for rank 4 TGFT

Proposition 4

$$\begin{aligned}\Gamma_{abcd} &= Z^2 M^{-1}_{abcd}\lambda_{6,1}\Big[\sum_p\Big(\frac{1}{M_{pbcd}-\Gamma_{pbcd}}-\frac{1}{M_{pbcd}-\Gamma_{pbcd}}\frac{\Gamma_{abcd}-\Gamma_{pbcd}}{Z(a^2-p^2)}\Big)\\ &\quad+\sum_{p,q,r}\frac{1}{M_{pqra}-\Gamma_{pqra}}\Big]\\ &+ Z^2 M^{-1}_{abcd}\lambda_{6,2}\Big[\sum_p\Big(\frac{1}{M_{pbcd}-\Gamma_{pbcd}}-\frac{1}{M_{pbcd}-\Gamma_{pbcd}}\frac{\Gamma_{abcd}-\Gamma_{pbcd}}{Z(a^2-p^2)}\Big)\\ &\quad+\sum_{p,q,r}\frac{1}{M_{pqra}-\Gamma_{pqra}}\Big]\\ &+ Z^2\lambda_{4,1}\Big[\sum_p\Big(\frac{1}{M_{pbcd}-\Gamma_{pbcd}}-\frac{1}{M_{pbcd}-\Gamma_{pbcd}}\frac{\Gamma_{abcd}-\Gamma_{pbcd}}{Z(a^2-p^2)}\Big)\\ &\quad+\sum_{p,q,r}\frac{1}{M_{pqra}-\Gamma_{pqra}}\Big]. \end{aligned} \tag{41}$$

To sum up, in this work

- We have presented a perturbative calculation of two-point correlation functions of rank 3 TGFT. As discussed earlier the correlation functions are given by combining Ward-Takahashi identities and Schwinger-Dyson equations that allows to establish the appropriate closed equation. The closed equation in the $4D$ case is also given.
- We proved that the nonperturbative techniques as developed in can be reported to the tensor situation. Indeed, although, we only solve our closed form equations for the two-point functions at initial orders, it is very promising to see that we can obtain even solutions in this highly combinatoric case.
- As future investigations, we can now undertake a calculation of the general solution at all orders of the coupling constants for both rank 3 and 4 models.
- Then we have studied in detail Landau like problem in a complete NC phase space.

Thank you for your attention