Dynamical systems approach to macroeconomics

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Really bad economics: hardcore (freshwater) DSGE

- The strand of DSGE economists associated with RBC theory made the following predictions after 2008:
  1. Increases government borrowing would lead to higher interest rates on government debt because of "crowding out".
  2. Increases in the money supply would lead to inflation.
  3. Fiscal stimulus has zero effect in a perfect world and negative effect in practice (because of decreased confidence).

Wrong prediction number 1

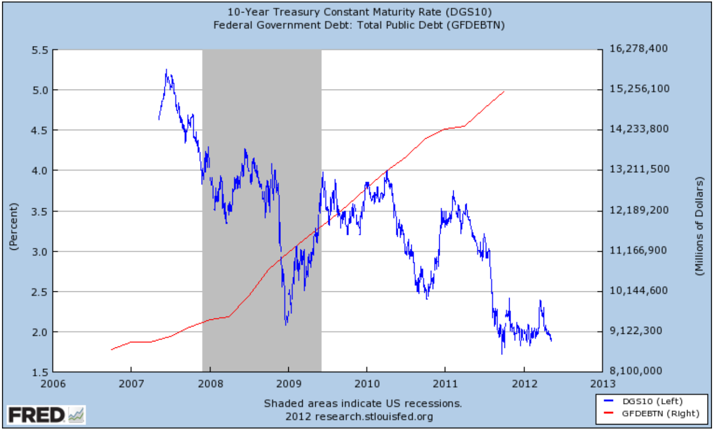


Figure: Government borrowing and interest rates.

Wrong prediction number 2

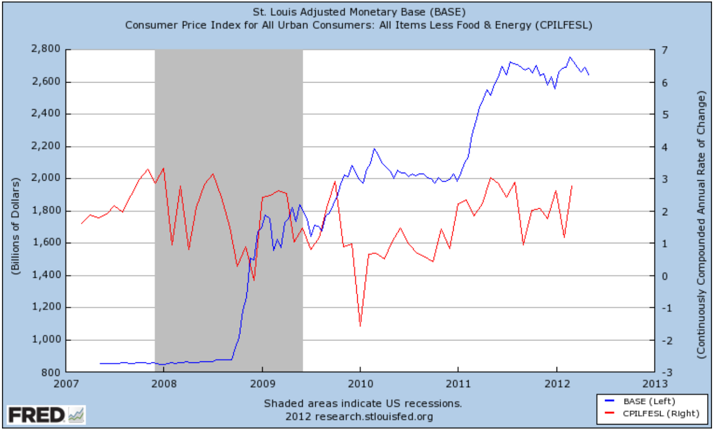


Figure: Monetary base and inflation.

Wrong prediction number 3

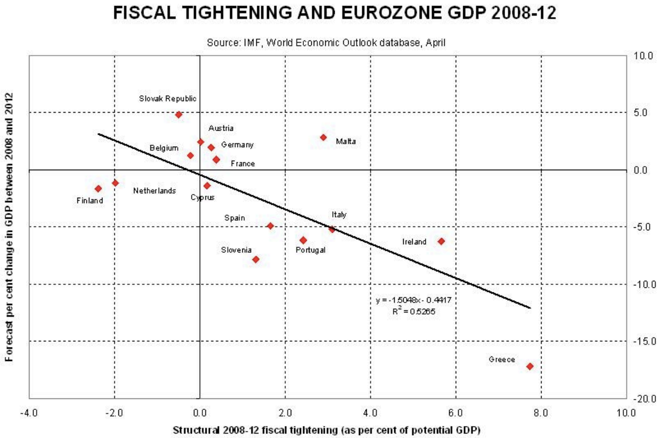


Figure: Fiscal tightening and GDP.

Better (but still bad) economics: soft core (saltwater) DSGE

- The strand of DSGE economists associated with New Keynesianism got all these predictions more or less right.
- Works by augmenting DSGE with 'imperfections' (sticky wages, asymmetric information, imperfect competition, frictions in financial markets, . . .).
- Still DSGE at core - analogous to adding epicycles to Ptolemaic planetary system.
- For example: "Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth – one person's liability is another person's asset." (Paul Krugman and Gauti B. Eggertsson, 2010, pp. 2-3)

Much better economics: SFC models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- Consider both real and monetary factors from the start
- Specify the balance sheet and transactions between sectors
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject silly (and mathematically unsound!) hypotheses such as the RARE individual (representative agent with rational expectations).
- See Godley and Lavoie (2007) for the full framework.

An example of a (fairly general) Godley table

	Households	Firms		Banks	Central Bank	Gov	Sum
Balance Sheet		current	capital				
Capital			$+K$				+K
Cash	$+H_h$		$+H_f$	$+H_b$	$-H$		0
Advances				$-A$	$+A$		0
Deposits	$+M_h$		$+M_f$	$-M$			0
Loans			$-L$	$+L$			0
Bills	$+B_h$		$+B_f$	$+B_b$	$+B_c$	$-B$	0
Equities	$+p_f E_f + p_b E_b$		$-p_f E_f$	$-p_b E_b$			0
Sum (net worth)	V_h		V_f	V_b	0	V_g	K
Transactions							
Consumption	$-C$	$+C$					0
Gov spending		$+G$				$-G$	0
Investment		$+I$	$-I$				0
memo [GDP]		[Y]					
Wages	$+W$	$-W$					0
Taxes	$-T_h$	$-T_f$		$-T_b$		$+T$	0
Interest on deposits	$+r_M M_h$	$+r_M M_f$		$-r_M M$			0
Interest on loans		$-r_L L$		$+r_L L - r_A A$	$+r_A A$		0
Interest on bills	$+r_B B_h$	$+r_B B_f$		$+r_B B_b$	$+r_B B_c$	$-r_B B$	0
Profits	$+F_{fd} + F_b$	$-F_f$	$+F_{fu}$	$-F_b$	$-F_c$	$+F_c$	0
Financial Balances	S_h	0	S_f	S_b	0	S_g	0
Flow of Funds							
Cash	$-\dot{H}_h$		$-\dot{H}_f$	$-\dot{H}_b$	$+\dot{H}$		0
Advances				$+\dot{A}$	$-\dot{A}$		0
Deposits	$-\dot{M}_h$		$-\dot{M}_f$	$+\dot{M}$			0
Loans			$+\dot{L}$	$-\dot{L}$			0
Bills	$-\dot{B}_h$		$-\dot{B}_f$	$-\dot{B}_b$	$-\dot{B}_c$	$+\dot{B}$	0
Equities	$-p_f \dot{E}_f - p_b \dot{E}_b$		$+p_f \dot{E}_f$	$+p_b \dot{E}_b$			0
Column sum	0	0	0	0	0	0	0

Another example: the Goodwin model

Balance Sheet	Households	Firms		Sum
Capital goods		$+K$		K
Sum (net worth)	V_h	V_f		K
Transactions		current	capital	
Consumption	$-C$	$+C$		0
Investment		$+I$	$-I$	0
Accounting memo [GDP]		$[Y]$		
Wages	$+W$	$-W$		0
Financial balances	0	Π_u	$-I$	0
Flow of Funds				
Capital goods		$+I$		I
Sum	0	Π_u		I
Change in net worth	0	$\Pi_u - \delta K$		$I - \delta K$

Deriving Goodwin

- Let $N = n_0 e^{\beta t}$ be the labour force, $a = a_0 e^{\alpha t}$ be its productivity and $\lambda = L/N$ be the employment rate.
- Define the total output $Y = aL$ and total capital as $K = \nu Y$.
- Assume that wages satisfy

$$\frac{dw}{dt} = \Phi(\lambda)w,$$

  where $\Phi(\lambda)$ is a Phillips curve.
- Let the wages share of total output be ω and profit share be $\pi = 1 - \omega$.
- Suppose further that the rate of new investment is given by

$$I = \frac{dK}{dt} = (1 - \omega)Y - \gamma K$$

Differential Equations

- It is easy to deduce that this leads to

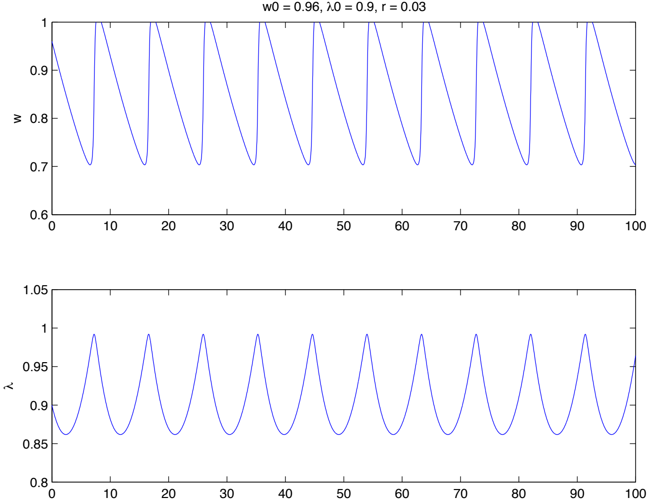
$$\frac{d\omega}{dt} = \omega(\Phi(\lambda) - \alpha) \tag{1}$$

$$\frac{d\lambda}{dt} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \gamma - \beta\right) \tag{2}$$

- This system is globally stable and leads to endogenous cycles of employment.

Example 1: basic Goodwin model

Example 1 (continued): basic Goodwin model



Yet another example: the Keen model

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Capital goods			$+K$		$+K$
Deposits	$+M_h$		$+M_f$	$-M$	0
Loans			$-L$	$+L$	0
Sum (net worth)	V_h		V_f	V_b	$+K$
Transactions					
Consumption	$-C$	$+C$			0
Investment		$+I$	$-I$		0
Accounting memo [GDP]		[Y]			
Wages	$+W$	$-W$			0
Interest on M	$+r_M M_h$	$+r_M M_f$		$-r_M M$	0
Interest on L		$-r_L L$		$+r_L L$	0
Profits		$-F_f$	$+F_{fu}$		0
Financial Balances	S_h	0	S_f	S_b	0
Flow of Funds					
Deposits	$-\dot{M}_h$		$-\dot{M}_f$	$+\dot{M}$	0
Loans			$+\dot{L}$	$-\dot{L}$	0
Column sum	0	0	0	0	0

Deriving Keen

- Consider the same model as before, but with a nonlinear investment function $I_g = \kappa(\pi_n/\nu)$ of the net profit share:

$$\pi_n = 1 - \omega - rd,$$

where $d = D/Y$ and the absolute debt level D evolves according to

$$\frac{dD}{dt} = I_g - \pi_n = rD + \kappa(\pi_n/\nu) - (1-\omega).$$

- We then find that

$$\frac{1}{Y}\frac{dY}{dt} = \mu(\omega, d), \tag{3}$$

where the growth rate taking into account the banking sector is now given by

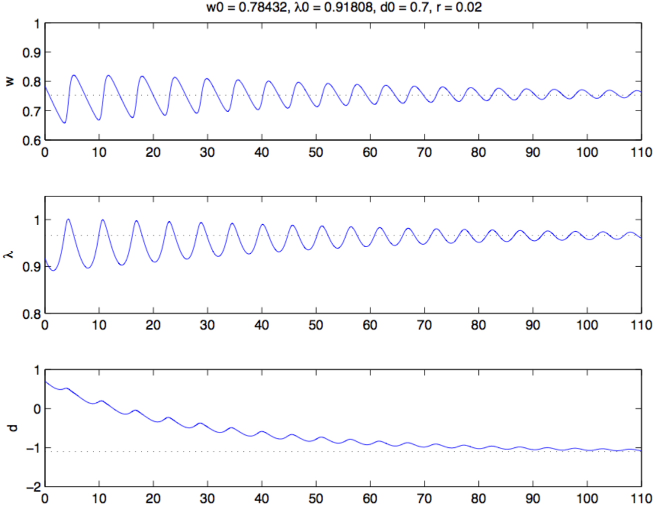
$$\mu(\omega, d) = \frac{\kappa\left(\frac{1-\omega-rd}{\nu}\right)}{\nu} - \gamma. \tag{4}$$

- The corresponding dynamical systems now reads

$$
\begin{aligned}
\frac{d\omega}{dt} &= \omega(\Phi(\lambda) - \alpha) \\
\frac{d\lambda}{dt} &= \lambda\left(\mu(\omega, d) - \alpha - \beta\right) \\
\frac{dd}{dt} &= d[r - \mu(\omega, d)] + \nu[\mu(\omega, d) + \gamma] - (1 - \omega)
\end{aligned}
$$

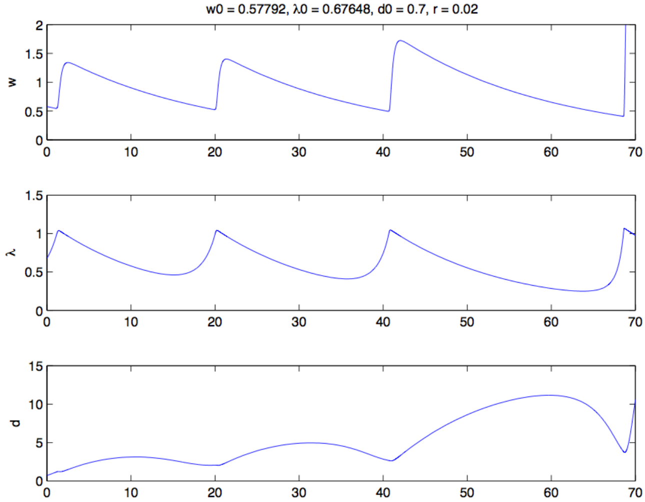
- This system is locally stable but globally unstable.

Example 2: convergent Keen model

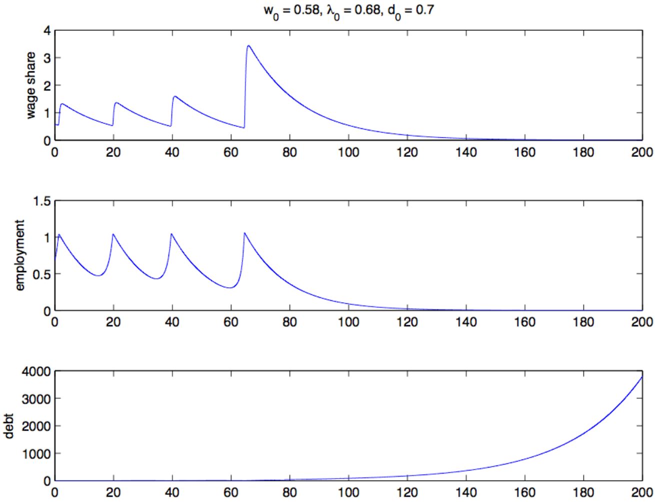


Example 2 (continued): convergent Keen model

Example 3: divergent Keen model

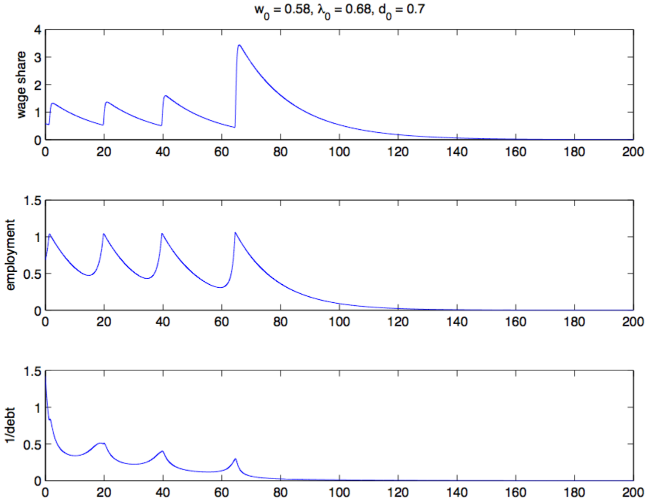


Example 3 (continued): divergent Keen model



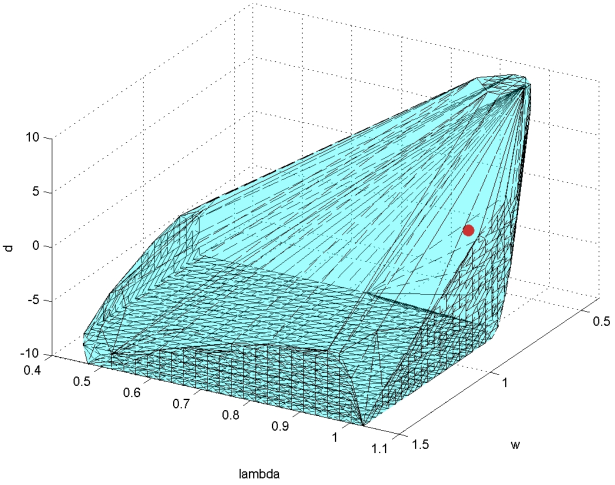
Example 3 (continued): divergent Keen model

Example 3 (continued): divergent Goodwin model with banks



Example 3 (continued): divergent Goodwin model with banks

Basin of convergence for Goodwin model with banks



Work in the project

- For a selection of models proposed in the 'formal Minsky' literature, the group will:
  1. Construct Godley tables and show stock-flow consistency.
  2. Write down the corresponding dynamical systems.
  3. Find equilibria and perform local stability analysis.
  4. Simulate the model and look for bifurcations, limit cycles, strange attractors, etc.
  5. Perform basic calibration to a sample dataset for OECD countries.