STABLE BIG BANG FORMATION IN SOLUTIONS TO THE EINSTEIN-SCALAR FIELD SYSTEM

JARED SPECK

Abstract

I will discuss some recent joint work with I. Rodnianski. We have shown that the well-known Friedmann-Lemaître-Robertson-Walker (FLRW) solution to the Einstein-scalar field system is non-linearly stable near $\Sigma_0 := \{t = 0\}$, where it has a Big Bang singularity. More precisely, we considered data on the Cauchy hypersurface $\Sigma_1 := \{t = 1\}$ that are close to the FLRW data as measured by a Sobolev norm. We studied the perturbed solution in the collapsing direction $t \downarrow 0$ and showed that it remains globally close to the FLRW solution for $t \in (0, 1]$. In particular, the perturbed solution has a Big Bang singularity along Σ_0 where its curvature blows up like t^{-2}. This proves Penrose's Strong Cosmic Censorship conjecture for the past half of the perturbed spacetimes.

From the point of view of analysis, the above results constitute a proof of stable blow-up for an open set of solutions to the Einstein-scalar field equations in an elliptic-hyperbolic gauge. The most important aspect of the proof is our identification of a new L^2-type *energy almost-monotonicity inequality* that holds for the solutions under consideration. I will highlight various aspects of the proof that are connected to the approximate monotonicity and that may be of general interest. In particular, I will exhibit two gauges in which the approximate monotonicity is visible: the well-known constant mean curvature-transported spatial coordinates gauge, and a related one-parameter family of gauges involving well-chosen parabolic PDEs for the lapse function.