Two-sided investments and matching with multi-dimensional cost types and attributes

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Investments and matching

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- sellers and buyers

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Main features

- investments affect the surplus/gains from trade that can be generated in future matches
- agents cannot bargain and contract with potential partners before they invest
- when agents choose investments, they take into account their costs and the payoff they expect to get in the matching market
- the prospect of competition provides incentives to invest

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Focus of the present paper

- economies with a competitive (continuum, frictionless) one-to-one matching market
- consequences of market incompleteness

A sketch of the model

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- continuum of heterogeneous buyers and sellers with quasi-linear utility functions: each agent is characterized by a cost type

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Two stages

- at stage 1, all agents simultaneously and non-cooperatively choose investments
- at stage 2, agents compete in a one-to-one matching market
  - sunk investments determine the match surplus
  - the market is an **assignment game**: matching is frictionless and utility is transferable $\Rightarrow$ based on their investments, buyers and sellers match efficiently

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Cole, Mailath and Postlewaite (2001a)

- investments are **one-dimensional** and match surplus is **supermodular**
- cost types are **one-dimensional** and cost functions are **submodular**

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- investment choices are not directed by a complete system of Walrasian payoffs for all ex-ante possible investments: there are **market** payoffs only for investments that exist at stage 2
- an agent who deviates to an otherwise non-existent investment can match with any marketed investment from the other side, leave the market payoff to the partner and keep the remaining surplus

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Cole, Mailath and Postlewaite (2001a)

- an efficient equilibrium always exists
- two examples of inefficient equilibria with coordination failures

Contributions (I)

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Motivation

- the sets of possible investments are multi-dimensional in most interesting environments
- multi-dimensional cost types are needed to model ex-ante heterogeneity
- general forms of surplus and cost functions

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I verify that efficient ex-post contracting equilibria exist in a general assignment game framework

Main contribution

- I shed light on **what enables/constrains/precludes** the existence of inefficient equilibria, both in environments with one-dimensional and with multi-dimensional heterogeneity

Contributions (II)

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Two kinds of inefficiency

- **inefficiency of joint investments**
- **mismatch** of buyers and sellers from an ex-ante perspective
  - cannot occur in the "1-d supermodular framework," where the matching of cost types must be positively assortative in any equilibrium

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Main contributions

- new sufficient condition for ruling out inefficiency of joint investments: "absence of technological multiplicity"
- analysis of mismatch in multi-dimensional environments without technological multiplicity
  - examples, require some insights from optimal transport
- new insights about the role of ex-ante heterogeneity for ruling out inefficiencies in environments with technological multiplicity

Related literature

Investments and matching

- Acemoglu (1996); Mailath, Postlewaite and Samuelson (2013)
- Peters and Siow (2002); Bhaskar and Hopkins (2013); Gall, Legros and Newman (2013)
- Chiappori, Iyigun and Weiss (2009); McCann, Shi, Siow and Wolthoff (2013)
- Cole, Mailath and Postlewaite (2001a,b); Felli and Roberts (2001)
- Nöldeke and Samuelson (2014)

Assignment games, optimal transport and hedonic pricing

- Shapley and Shubik (1971); Becker (1973); Gretzky, Ostroy and Zame (1992, 1999)
- Villani (2009)
- Rosen(1974); Ekeland (2005, 2010); Chiappori, McCann and Nesheim (2010)

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  - characterized by **cost types** $b \in B$ and $s \in S$
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  - cost functions $c_B(x, b)$ and $c_S(y, s)$
- B, S, X and Y are compact metric spaces
- $v : X \times Y \to \mathbb{R}_+$, $c_B : X \times B \to \mathbb{R}_+$ and $c_S : Y \times S \to \mathbb{R}_+$ are continuous
- for simplicity: unmatched agents create zero surplus

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- the heterogeneous ex-ante populations of buyers and sellers are described by probability measures μ_B on B and μ_S on S
- μ_B, μ_S, v, c_B and c_S are common knowledge at stage 1

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The possible **matchings** of μ_X and μ_Y are the measures π_2 on $X \times Y$ with marginal measures μ_X and μ_Y: $\pi_2 \in \Pi(\mu_X, \mu_Y)$

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- core payoff functions $\psi_X^* : \text{Supp}(\mu_X) \to \mathbb{R}$ and $\psi_Y^* : \text{Supp}(\mu_Y) \to \mathbb{R}$
  - $\psi_Y^*(y) + \psi_X^*(x) = v(x, y)$ on $\text{Supp}(\pi_2^*)$
  - for all $(x, y) \in \text{Supp}(\mu_X) \times \text{Supp}(\mu_Y)$: $\psi_Y^*(y) + \psi_X^*(x) \geq v(x, y)$

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- ψ_X^* and ψ_Y^* are continuous

Stage 1: Best replies

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In **ex-post contracting equilibrium**, agents' attribute choices must "best-reply" to the correctly anticipated trading possibilities and the equilibrium outcome $(\pi_2^*, \psi_X^*, \psi_Y^*)$ of the endogenous market (μ_X, μ_Y, v) that results from others' sunk investments. In particular,

- if $x \in \text{Supp}(\mu_X)$ is an equilibrium investment of type b, then x must satisfy

$$\psi_X^*(x) - c_B(x, b) = \max_{x' \in X, y \in \text{Supp}(\mu_Y)} (v(x', y) - \psi_Y^*(y) - c_B(x', b))$$

- if $y \in \text{Supp}(\mu_Y)$ is an equilibrium investment of type s, then y must satisfy

$$\psi_Y^*(y) - c_S(y, s) = \max_{y' \in Y, x \in \text{Supp}(\mu_X)} (v(x, y') - \psi_X^*(x) - c_S(y', s))$$

Ex-post contracting equilibrium

Formal definition

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Definition

An **ex-post contracting equilibrium** is a tuple $((\beta, \sigma, \pi_1), (\pi_2^*, \psi_X^*, \psi_Y^*))$, in which (β, σ, π_1) is a regular investment profile and $(\pi_2^*, \psi_X^*, \psi_Y^*)$ is a stable and feasible bargaining outcome for (μ_X, μ_Y, v), such that for all $(b, s) \in \text{Supp}(\pi_1)$ it holds:

$$\psi_X^*(\beta(b,s)) - c_B(\beta(b,s), b) = \max_{x' \in X, y \in \text{Supp}(\mu_Y)} (v(x', y) - \psi_Y^*(y) - c_B(x', b)) =: r_B(b),$$

$$\psi_Y^*(\sigma(b,s)) - c_S(\sigma(b,s), s) = \max_{y' \in Y, x \in \text{Supp}(\mu_X)} (v(x, y') - \psi_X^*(x) - c_S(y', s)) =: r_S(s).$$

The efficiency benchmark

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The **maximal net surplus** that a pair (b, s) can generate is

$$w(b, s) = \max_{x \in X, y \in Y} v(x, y) - c_B(x, b) - c_S(y, s)$$

- **jointly optimal** attributes $(x^*(b, s), y^*(b, s))$ exist for all (b, s)
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The stable outcomes $(\pi_1^*, \psi_B^*, \psi_S^*)$ of the assignment game (μ_B, μ_S, w) provide the benchmark of ex-ante efficiency

- they describe how agents would match and divide net surplus if buyers and sellers could bargain in a frictionless market and write complete contracts before they invest, so that partners choose jointly optimal attributes

Efficient equilibria

Efficient equilibria

Result

Every stable outcome $(\pi_1^*, \psi_B^*, \psi_S^*)$ of (μ_B, μ_S, w) can be supported by an ex-post contracting equilibrium. In particular, an efficient equilibrium exists.

Two manifestations of inefficiency

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Buyers and sellers may be **mismatched** from an ex-ante perspective

- the matching of cost types that is associated with the equilibrium investment behavior and the matching of attributes is not efficient for the benchmark assignment game (μ_B, μ_S, w)

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- the matching of cost types that is associated with the equilibrium investment behavior and the matching of attributes is not efficient for the benchmark assignment game (μ_B, μ_S, w)

There may be **inefficiency of joint investments**

- agents' attributes are not jointly optimal in a strictly positive mass of matches that arise in equilibrium

Full appropriation games

Full appropriation games

Consider the following complete information, "full appropriation" (FA) game between a buyer of type b and a seller of type s

- strategy spaces are X and Y
- payoffs are $v(x, y) - c_B(x, b)$ and $v(x, y) - c_S(y, s)$

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Lemma

The attributes of a buyer of type b and a seller of type s who are matched in equilibrium must be a Nash equilibrium (NE) of the FA game between them.

Technological multiplicity

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Proposition

Assume that for all $b \in \text{Supp}(\mu_B)$ and $s \in \text{Supp}(\mu_S)$, the FA game between b and s has a unique NE. Then ex-post contracting equilibria cannot feature inefficiency of joint investments.

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Note

- jointly optimal attributes $x^*(b,s)$ and $y^*(b,s)$ are always a NE of the FA game between b and s, as they maximize $v(x,y) - c_B(x,b) - c_S(y,s)$

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Definition

An environment displays **technological multiplicity** if FA games have more than one pure strategy NE for some $(b, s) \in \text{Supp}(\mu_B) \times \text{Supp}(\mu_S)$.

The 1-d supermodular framework

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Condition (1dS)

Let $X \setminus \{x_\varnothing\}, Y \setminus \{y_\varnothing\}, B \setminus \{b_\varnothing\}, S \setminus \{s_\varnothing\} \subset \mathbb{R}_+$. Assume that v is strictly supermodular in (x, y), c_B is strictly submodular in (x, b), and c_S is strictly submodular in (y, s).

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Lemma

Let Condition 1dS hold. Then the induced matching of buyer and seller cost types is positively assortative in every ex-post contracting equilibrium. Mismatch is impossible.

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Lemma

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- equilibrium attribute choices are increasing in type
  - an equilibrium attribute x of type b must belong to $\text{argmax}_{x' \in X} \left(\max_{y \in \text{Supp}(\mu_Y)} (v(x', y) - \psi_Y^*(y)) - c_B(x', b) \right)$
- the matching of attributes is positively assortative

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- marketed attributes $x^*(b,s)$ are attractive targets for deviations by agents s' not too different from s, similarly for $y^*(b,s)$ and buyers b'

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- these requirements constrain mismatch if there is some differentiation of agents ex-ante

Mismatch and its constraints in a 2-d bilinear model (I)

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The standard bilinear model

Let $\text{Supp}(\mu_B) \setminus \{b_\varnothing\} \subset \mathbb{R}^2_+ \setminus \{0\}$, $\text{Supp}(\mu_S) \setminus \{s_\varnothing\} \subset \mathbb{R}^2_+ \setminus \{0\}$ and $X \setminus \{x_\varnothing\} = Y \setminus \{y_\varnothing\} = \mathbb{R}^2_+$. Surplus and costs are given by $v(x, y) = x \cdot y = x_1 y_1 + x_2 y_2$, $c_B(x, b) = \frac{x_1^4}{b_1^2} + \frac{x_2^4}{b_2^2}$ and $c_S(y, s) = \frac{y_1^4}{s_1^2} + \frac{y_2^4}{s_2^2}$.

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- FA games have unique non-trivial NE, given by

$$(x^*(b, s), y^*(b, s)) = \frac{1}{2}\left(\left(b_1^{\frac{3}{4}} s_1^{\frac{1}{4}}, b_2^{\frac{3}{4}} s_2^{\frac{1}{4}}\right), \left(b_1^{\frac{1}{4}} s_1^{\frac{3}{4}}, b_2^{\frac{1}{4}} s_2^{\frac{3}{4}}\right)\right)$$

Mismatch and its constraints in a 2-d bilinear model (I)

The standard bilinear model

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Mismatch and its constraints in a 2-d bilinear model (II)

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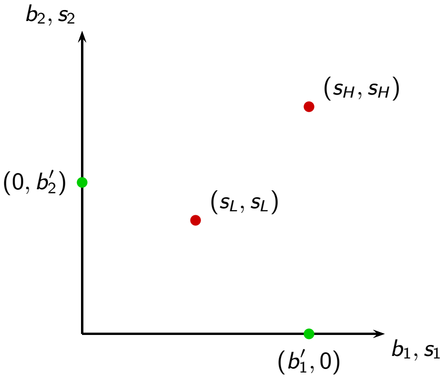
Let $\mu_S = a_H \delta_{(s_H, s_H)} + (1 - a_H)\delta_{(s_L, s_L)}$, where $0 < s_L < s_H$ and $0 < a_H < 1$. Moreover, $\mu_B = a_1 \delta_{(b_1', 0)} + a_2 \delta_{(0, b_2')} + (1 - a_1 - a_2)\delta_{b_\varnothing}$, where $0 < a_1, a_2, b_1', b_2'$ and $a_1 + a_2 < 1$. Finally, let $b_1' > b_2'$ and $a_H < a_1 + a_2$.



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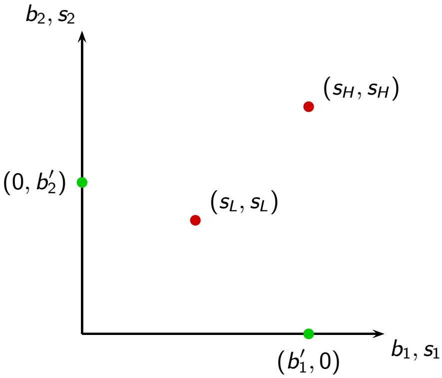

- $w((b_1, b_2), (s_1, s_1)) = \frac{1}{8}(b_1 + b_2)s_1$
- the ex-ante efficient matching is positively assortative in s_1 and $b_1 + b_2$

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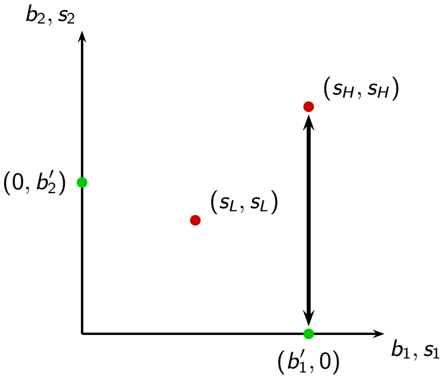

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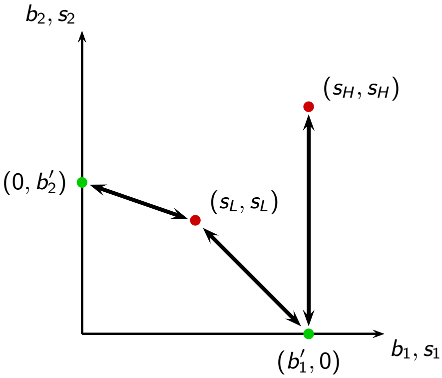

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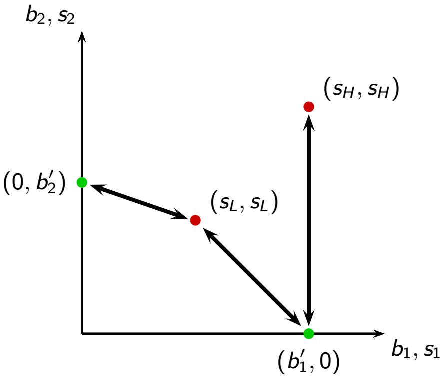

- $w((b_1,b_2),(s_1,s_1)) = \frac{1}{8}(b_1+b_2)s_1$
- the ex-ante efficient matching is positively assortative in s_1 and b_1+b_2
- depicted case: $a_H < a_1$
- attributes in the endogenous market: $x^*((b_1',0),(s_H,s_H))$, $x^*((b_1',0),(s_L,s_L))$, $x^*((0,b_2'),(s_L,s_L))$, $y^*((b_1',0),(s_H,s_H))$, $y^*((b_1',0),(s_L,s_L))$, $y^*((0,b_2'),(s_L,s_L))$
- e.g. $x^*((b_1',0),(s_H,s_H)) = \left(\frac{1}{2}b_1'^{\frac{3}{4}}s_H^{\frac{1}{4}}, 0\right)$, $y^*((b_1',0),(s_H,s_H)) = \left(\frac{1}{2}b_1'^{\frac{1}{4}}s_H^{\frac{3}{4}}, 0\right)$

Mismatch and its constraints in a 2-d bilinear model (III)

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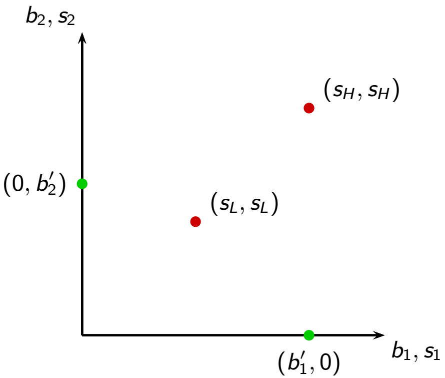
Claim

Consider the environment of Example 1. If $a_H < a_2$, then there is exactly one additional, mismatch inefficient equilibrium if and only if $\frac{2}{3}\frac{b_2'}{b_1'} \geq \frac{\left(\frac{s_H}{s_L}\right)^{\frac{2}{3}}-1}{\frac{s_H}{s_L}-1}$. Otherwise, only the ex-ante efficient equilibrium exists.



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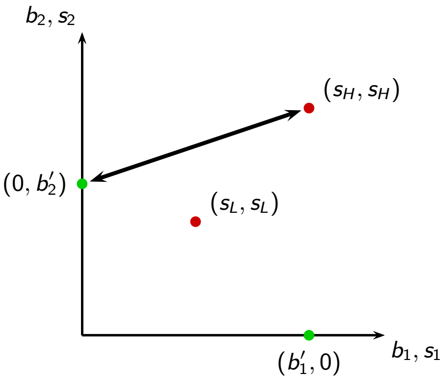

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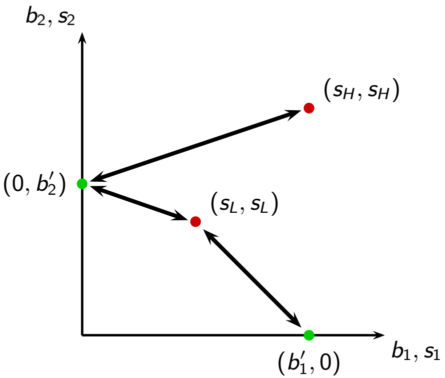

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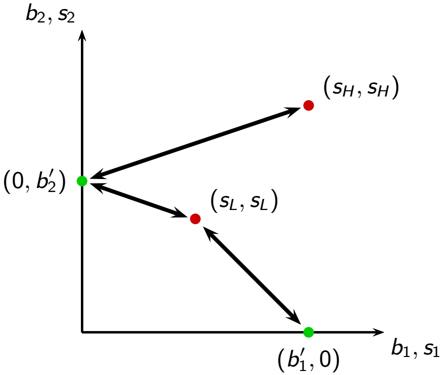

- attributes in the endogenous market: $x^*((b_1', 0), (s_L, s_L))$, $x^*((0, b_2'), (s_H, s_H))$, $x^*((0, b_2'), (s_L, s_L))$, $y^*((b_1', 0), (s_L, s_L))$, $y^*((0, b_2'), (s_H, s_H))$, $y^*((0, b_2'), (s_L, s_L))$

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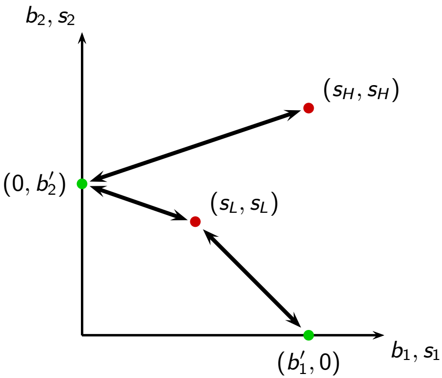

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- (s_H, s_H)-sellers have no incentive to deviate by investing optimally for a match with $x^*((b_1',0),(s_L,s_L))$ if and only if the condition of the Claim holds

Mismatch and its constraints in a 2-d bilinear model (IV)

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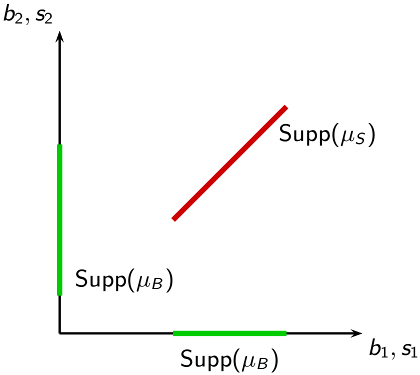
Example 2

$\text{Supp}(\mu_S) = \{(s_1, s_1) | s_L \leq s_1 \leq s_H\}$, for some $s_L < s_H$. μ_B is compactly supported in the union of $(\mathbb{R}_+ \setminus \{0\}) \times \{0\}$, $\{0\} \times (\mathbb{R}_+ \setminus \{0\})$ and $\{b_\varnothing\}$. The restrictions of μ_B to $(\mathbb{R}_+ \setminus \{0\}) \times \{0\}$ and $\{0\} \times (\mathbb{R}_+ \setminus \{0\})$ have interval support.



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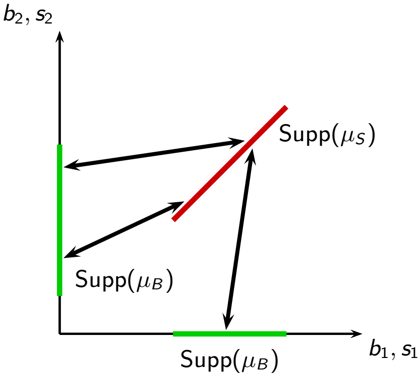

- result: the only ex-post contracting equilibrium is the ex-ante efficient one
- cost types are matched positively assortatively in s_1 and $b_1 + b_2$

Mismatch and its constraints in a 2-d bilinear model (V)

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Remarks

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- in Examples 1 and 2, results from the theory of assortative matching can be used to identify the efficient matching and to evaluate whether inefficient equilibria exist

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- this is not feasible in more complex environments

Characterization from optimal transport

- a matching $\pi_1 \in \Pi(\mu_B, \mu_S)$ is efficient if and only if it is concentrated on a w-cyclically monotone set

Definition

A set $A \subset B \times S$ is called **w-cyclically monotone** if for all $K \in \mathbb{N}$, $(b_1, s_1), ..., (b_K, s_K) \in A$ and $s_{K+1} = s_1$, the following inequality is satisfied.

$$\sum_{i=1}^{K} w(b_i, s_i) \geq \sum_{i=1}^{K} w(b_i, s_{i+1}).$$

Mismatch and its constraints in a 2-d bilinear model (VI)

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Theorem (Villani)

Let $\mathrm{Supp}(\mu_B), \mathrm{Supp}(\mu_S) \subset (\mathbb{R}_+ \setminus \{0\})^2$ be closures of bounded, open and uniformly convex sets with smooth boundaries. Assume that μ_B and μ_S admit smooth, strictly positive densities on $\mathrm{Supp}(\mu_B)$ and $\mathrm{Supp}(\mu_S)$. Then, the stable outcomes $(\pi_1^*, \psi_B^*, \psi_S^*)$ of (μ_B, μ_S, w) satisfy:

- ψ_B^* and ψ_S^* are smooth, and unique up to an additive constant,
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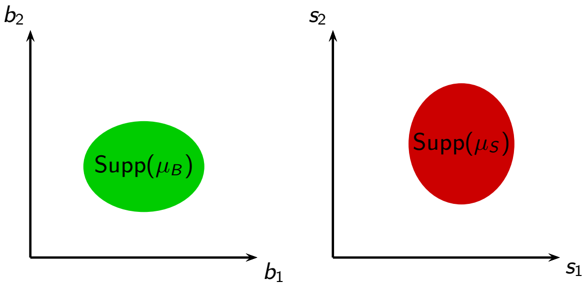

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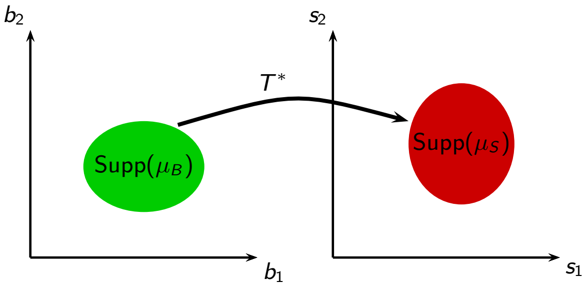

Mismatch and its constraints in a 2-d bilinear model (VII)

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Theorem

Consider the environment of Theorem (Villani), and assume in addition that $\left(\frac{s_1}{b_1}\frac{b_2}{s_2} + \frac{s_2}{b_2}\frac{b_1}{s_1}\right) < 32$ for all $b \in \text{Supp}(\mu_B)$, $s \in \text{Supp}(\mu_S)$. If $T : \text{Supp}(\mu_B) \to \text{Supp}(\mu_S)$ is a smooth matching of buyer and seller types that is compatible with an ex-post contracting equilibrium, then T is ex-ante efficient.

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- hence, the matching T of buyer and seller types associated with the equilibrium is concentrated on the subdifferential of a convex function
- for bilinear w, this is a w-cyclically monotone set $\Rightarrow$ T is efficient

Environments with technological multiplicity

An under-investment example à la (CMP) (I)

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Example 3

Let $v(x, y) = \max\left(xy, \frac{1}{2}x^{\frac{3}{2}}y^{\frac{3}{2}}\right)$, $c_B(x, b) = \frac{x^4}{b^2}$ and $c_S(y, s) = \frac{y^4}{s^2}$. μ_B and μ_S have interval support. For simplicity, $\mu_B = \mu_S$.

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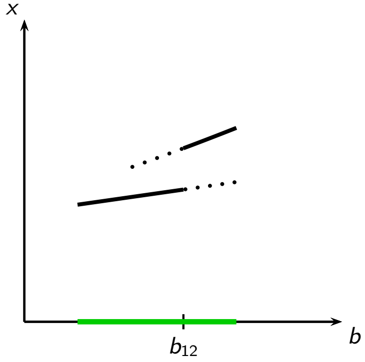

The efficient equilibrium

- v has two regimes of complementarity
- $x^*(b, s = b)$ and $y^*(b, s = b)$ jump from $\frac{b}{2}$ to $\frac{3b^2}{16}$ at $b = b_{12}$

Environments with technological multiplicity

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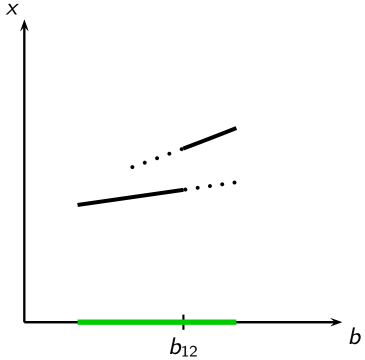

The efficient equilibrium

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- $x^*(b, s = b)$ and $y^*(b, s = b)$ jump from $\frac{b}{2}$ to $\frac{3b^2}{16}$ at $b = b_{12}$
- however, attributes $\left(\frac{b}{2}, \frac{b}{2}\right)$ remain a NE of the FA game between b and $s = b$ for $b > b_{12}$

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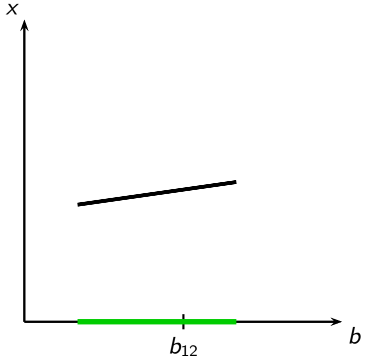


The under-investment equilibrium

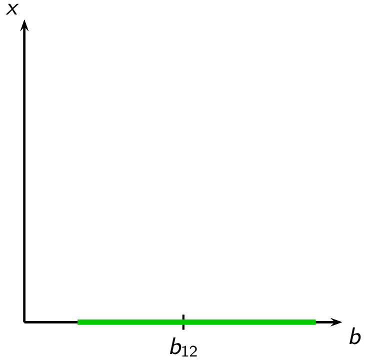
- this enables an equilibrium in which types with lower costs than the "indifference type" b_{12} under-invest, unless...

Environments with technological multiplicity

An under-investment example à la (CMP) (II)

Example 3

Let $v(x, y) = \max\left(xy, \frac{1}{2}x^{\frac{3}{2}}y^{\frac{3}{2}}\right)$, $c_B(x, b) = \frac{x^4}{b^2}$ and $c_S(y, s) = \frac{y^4}{s^2}$. μ_B and μ_S have interval support. For simplicity, $\mu_B = \mu_S$.



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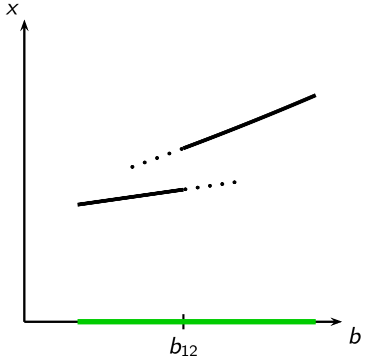

The efficient equilibrium

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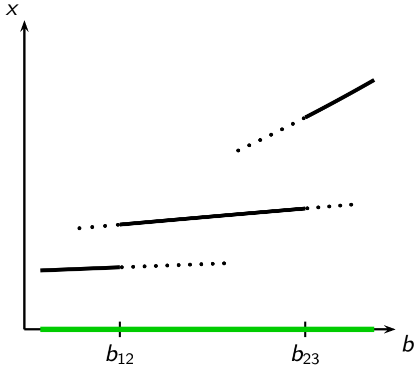

The efficient equilibrium

- v has three regimes of complementarity

Environments with technological multiplicity

Simultaneous under- and over-investment: the case of missing middle sectors (II)

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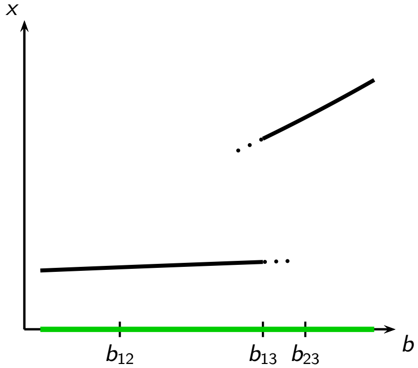

An inefficient equilibrium

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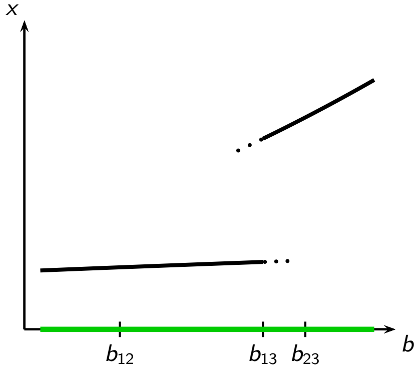

An inefficient equilibrium

- even extreme exogenous heterogeneity does not rule out the inefficient equilibrium

Conclusion

Take home messages

- technological multiplicity is the key source of potential inefficiencies
- even extreme ex-ante heterogeneity may be insufficient for ruling out inefficient equilibria