McMaster University

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THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

DIRECTOR'S SEMINAR

SPEAKER:

LUDWIG REICH
University of Graz

On the Topic:

"On Families of Commuting Formal Power Series Transformations and Iteratives Roots"

Let $\mathcal{F}=(F_\alpha)_{\alpha\in I}$ be a family of formal power series $F_\alpha(x)=\rho_\alpha x+c_2^{(\alpha)}x^2+\dots$, with $\rho_\alpha\neq 0$ for all α. This is a family of commuting series if $F_\alpha\circ F_\beta=F_\beta\circ F_\alpha$ for all $\alpha,\beta\in I$, where $\circ$ denotes substitution. Moreover assume that $\mathcal{F}$ is maximal with respect to inclusion. Then it is known that there is a close connection between these families and the so called Aczél-Jabotinsky differential equations of the third kind

(AJ) $$(G\circ\phi)(x)=\frac{d\phi}{dx}\cdot G(x)$$

in the following way: To each maximal family $\mathcal{F}$ there exists exactly one series $G(x)=x^m+d_{m+1}x^{m+1}+\dots, m\geq 1$, the generator of $\mathcal{F}$, such that $\mathcal{F}$ is the set of all solutions $\phi(x)=\rho x+c_2x^2+\dots,\rho\neq 0$, of (AJ) formed with the generator $G(x)$. From this fact we conclude detailed informations about the structure of $\mathcal{F}$ which is clearly an abelian group.
In our talk we are going to apply these resutls for studying Babbage's functional equation

(B) $$\phi^N=F$$

for a given series $F(x)=\sigma x+d_2x^2+\dots,\sigma\neq 0$, and a given $N\in\mathbb{N}$. Here ϕ^N means the N-th iterate of ϕ and ϕ is unknown. We will present criteria for the existence of solutions ϕ of (B) and then describe the general solution in a constructive way.

Monday, June 21, 1993

4:00 pm, room 3018

at

The Fields Institute

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